AP Calculus AB

Name: **Mrs. Rivero**

Limits & Continuity Assessment

Instructions: Show work or provide an explanation. Calculators are not permitted. Good luck and have fun-ctions. May the limit of your grade be positive infinity!

1) Find $\lim \dfrac{\sqrt{x+2}-\sqrt{2}}{x}$ Show all work. (4 points)

$$\lim_{x \to 0} \left(\frac{\sqrt{x+2}-\sqrt{2}}{x} \right) \left(\frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}} \right) =$$

$$= \lim_{x \to 0} \frac{x+2-2}{x(\sqrt{x+2}+\sqrt{2})}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{x+2}+\sqrt{2}}$$

$$\frac{1}{\sqrt{0+2}+\sqrt{2}}$$

$$= \boxed{\frac{1}{2\sqrt{2}}}$$

2) Let $F(x) = \begin{cases} \dfrac{x^2-5x-6}{x-6}, & x \ne 6 \\ 3k+2, & x = 6 \end{cases}$

a) Find $\lim_{x \to 6} F(x)$ Show all proper steps. (1 points)

$$\lim_{x \to 6} \frac{(x-6)(x+1)}{x-6}$$

$$\lim_{x \to 6} x+1 = 7$$

b) Find the value of k such that $\lim_{x \to 6} F(x) = F(6)$ Show all work. (2 points)

$$3k+2 = 7$$
$$3k = 5$$
$$k = \frac{5}{3}$$

3. (4 points)

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

Show that f is continuous at $x = 0$.

① $\lim\limits_{x \to 0^-} 1 - 2\sin x = 1$

$\lim\limits_{x \to 0^+} e^{-4x} = 1$

$\left.\right\} \lim\limits_{x \to 0} f(x) = 1$

③ Since $\lim\limits_{x \to 0} f(x) = f(0)$,

$f(x)$ is continuous at $x = 0$.

② $f(0) = 1 - 2\sin 0$

$f(0) = 1$

4. (4 points) Mrs. Rivero drops her English book off of the top of a 220-foot building. The position function of the book is given by the following equation

$$s(t) = -16t^2 + 220.$$

Find the velocity of the book when $t = 1$ second. Be sure to include units and to show all steps.

inst velocity

$M_{tan} = \lim\limits_{t \to 1} \dfrac{s(1+h) - s(1)}{h}$

$= \lim\limits_{t \to 1} \dfrac{-16(1+h)^2 + 220 - (-16 \cdot 1^2 + 220)}{h}$

$= \lim\limits_{t \to 1} \dfrac{-16(1 + 2h + h^2) + 220 - (-16 + 220)}{h}$

$\to \lim\limits_{t \to 1} \dfrac{-16 - 32h - 16h^2 \cdot \cancel{220} \cancel{+16} \cancel{-220}}{h}$

$\lim\limits_{t \to 1} -32 - 16h = \boxed{-32 \dfrac{ft}{sec}}$

5. (3 points) List the three requirements for a function (f(x)) to be continuous at a point (say at x=c).

① $\lim\limits_{x \to c} f(x)$ exists

② $f(c)$ exists

③ $\lim\limits_{x \to c} f(x) = f(c)$

4 points each
Evaluate each limit.

6. $\lim_{x \to 2} \dfrac{x^2 - 7x + 10}{x^2 - 4}$ $\dfrac{4(4+10)}{0} = \dfrac{0}{0}$

$\lim_{x \to 2} \dfrac{(x-5)(x-2)}{(x+2)(x-2)}$

$\lim_{x \to 2} \dfrac{x-5}{x+2} = \boxed{-\dfrac{3}{4}}$

7. $\lim_{x \to 0} \dfrac{(x+3)^3 - 27}{x}$ $\dfrac{0}{0}$

$\lim_{x \to 0} \dfrac{x^3 + 9x^2 + \overset{27x}{\tfrac{27}{?}} + 27 - 27}{x}$

$\lim_{x \to 0} \dfrac{x^3 + 9x^2 + 27x}{x}$

$\lim_{x \to 0} x^2 + 9x + 27 = \boxed{27}$

8. $\lim_{x \to \infty} \dfrac{4x^3 + x}{x^2 - x^3} = -4$

C

9. $\lim_{x \to -3} \dfrac{x+3}{x^2 - 7x + 12}$

$\lim_{x \to -3} \dfrac{(x+3)}{(x-4)(x-3)}$ $\dfrac{0}{-7 \cdot -6} = 0$

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10 (2 points) If $f(3) = -2$, what is the limit of $f(x)$ as x approaches 3? Explain,
using complete sentence(s)   You cannot evaluate $\lim_{x \to 3} f(x)$.

$f(3) = -2$ doesn't have anything to do with ↑.

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11. (2 points) If the limit of $g(x)$ as x approaches -2 is 5, what is $g(-2)$? Explain,
using complete sentence(s)

$\lim_{x \to -2} g(x) = 5$ tells us nothing about $g(-2)$.

The limit is the intended height of a function.
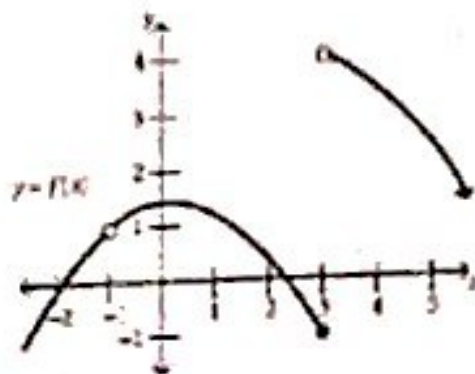
12 One point each; no work or explanations needed.

Figure 5.5

$y = f(x)$

1. $\lim\limits_{x \to -1^-} f(x) = 1$

2. $\lim\limits_{x \to -1^+} f(x) = 1$

3. $\lim\limits_{x \to -1} f(x) = 1$

4. $f(-1)$ dne

5. Is f continuous at $x = -1$? No

6. $\lim\limits_{x \to 3^-} f(x) = -1$

7. $\lim\limits_{x \to 3^+} f(x) = 4$

8. $\lim\limits_{x \to 3} f(x)$ dne

9. $f(3) = -1$

10. Is f continuous at $x = 1$? Yes

(4 points) 13   Compute $\lim\limits_{h \to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$, or indicate that the limit doesn't exist.

$$\lim\limits_{h \to 0} \frac{\dfrac{3 - (3+h)}{(3+h)(3)}}{h}$$

$$\lim\limits_{h \to 0} \frac{h}{3(3+h) \cdot h}$$

$$\lim\limits_{h \to 0} \frac{1}{3(3+h)} \quad \boxed{\dfrac{1}{9}}$$