AP Calculus AB

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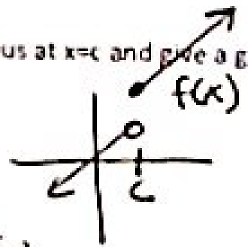
Test Review Limits

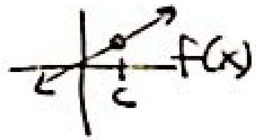
Instructions: Be sure to follow the directions for each section. SHOW WORK! Good luck and have fun-ctions! Calculators are permitted on this test.

(6 points) 1. List the three requirements for a function to be continuous at x=c and give a graphical example of a function failing to be continuous for each reason

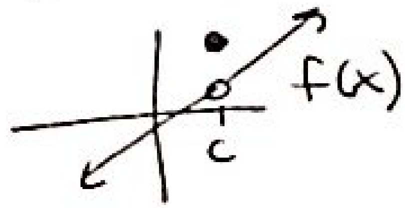
① $\lim_{x \to c} f(x)$ exists ⟶



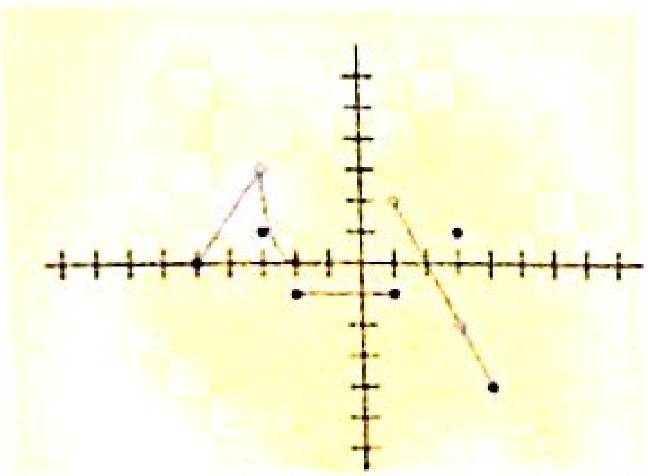
② f(c) exists ⟶



③ $\lim_{x \to c} f(x) = f(c)$ ⟶



2. (1 point each) Using the graph, evaluate each. (Scale is by 1's)

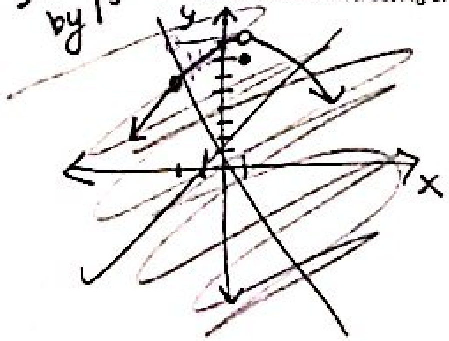


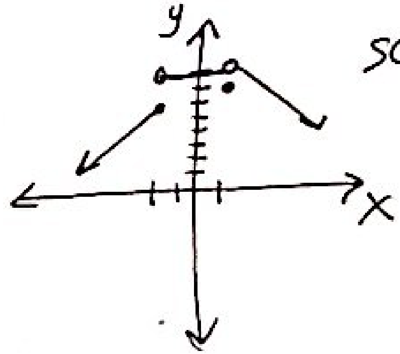
a. $\lim_{x \to 3} g(x)$ -2	b. $\lim_{x \to 1} g(x)$ dne
c. $\lim_{x \to 0} g(x)$ -1	d. $\lim_{x \to -2^-} g(x)$ 0
e. $\lim_{x \to -3} g(x)$ 3	f. $\lim_{x \to 4} g(x)$ -4
g. $\lim_{x \to 1^+} g(x)$ 2	h. $\lim_{x \to 2} g(x)$ 0
i. $\lim_{x \to 1^-} g(x)$ -1	j. $\lim_{x \to -2^+} g(x)$ -1

3. (5 points) Sketch an example of a *function* with the given attributes.

- $f(1)=7$
- $f(-2)=6$
- The limit as x approaches -2 of f(x) does not exist; however, both one-sided limits exist.
- The limit as x approaches 1 of f(x) is equal to 8.
- The function is decreasing on $(1,\infty)$

scale by 1's





scale by 1's

4. (6 points) Consider the following function:

$$f(x) = \begin{cases} a + bx, & if\ x > 2 \\ 3, & if\ x = 2 \\ b - ax^2, & if\ x < 2 \end{cases}$$

Determine the values of constants a and b so that $\lim_{x\to 2} f(x)$ exists and is equal to $f(2)$.

$f(2)=3$

$a+2b=3$

$-2(b-4a=3)$

$-2b+8a=-6$

$9a=-3$

$a=-\frac{1}{3}$ ☆

$a+2b=3$

$-\frac{1}{3}+2b=3$

$2b=\frac{10}{3}$

$b=5/3$ ☆

5. (8 points) Find the equation of the tangent line to $f(x) = 14 - 3x^2$ at $x=-2$. Show all calculations and include a graph. Use the following equation $m_{tan} = \lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h}$

$$m_{tan} = \lim_{h \to 0} \frac{f(-2+h) - f(-2)}{h}$$

$$= \lim_{h \to 0} \frac{14-3(-2+h)^2 - [14-3(-2)^2]}{h}$$

$$= \lim_{h \to 0} \frac{14-3(4-4h+h^2)-(14-3\cdot 4)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{14} - \cancel{12} + 12h - 3h^2 - \cancel{14} + \cancel{12}}{h}$$

$$= \lim_{h \to 0} \frac{h(12-3h)}{h}$$

$$\lim_{h \to 0} 12 - 3h = 12$$

6. (4 points) Let $f(x) = x^2 + 2x$. What is the average rate of change of $f(x)$ on the interval [1,3]?

$$= \frac{f(3) - f(1)}{3-1}$$

$$= \frac{9+6-(1^2+2\cdot 1)}{2}$$

$$= \frac{15-(3)}{2}$$

$$= 6$$

7. (2 points) If the limit as x approaches 2 is 4, what does that tell you about f(2)? Explain.

Nothing. A limit is the intended height of a function. $\lim_{x \to c} f(x)$ doesn't have to equal $f(c)$.

8. (2 points) If f(2) = 4, what does that tell you about the limit of f(x) as x approaches 2? Explain.

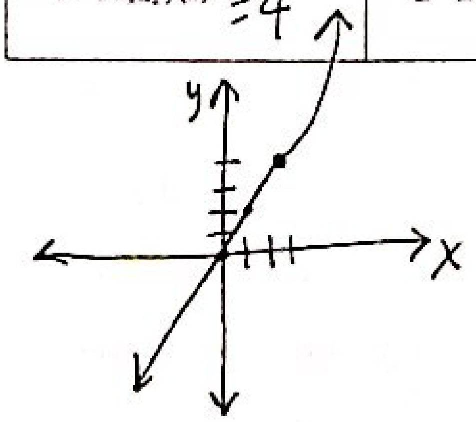
9. (2 points) Write, in words, how to say this. $\lim_{x \to 3} f(x) = 4$

The limit as x approaches 3 of f(x) is 4.

10. (12 points) Sketch the graph of $f(x) = \begin{cases} 2x, & x < 2 \\ x^2, & x \geq 2 \end{cases}$ and identify each limit. (Be sure to include a scale)

a. $\lim_{x \to 2^-} f(x)$ $= 4$	b. $\lim_{x \to 2^+} f(x)$ $= 4$
c. $\lim_{x \to 2} f(x)$ $= 4$	d. $\lim_{x \to 1} f(x)$ 2

scale by 1's



11. (8 points) If an object travels a distance of $s = 2t^2 - 5t + 1$, where s is in feet and t is in seconds, find

(a) the average velocity of the object within the first 10 seconds. Show all calculations and include proper units.

$$= \frac{s(10) - s(0)}{10 - 0}$$

$$= \frac{2 \cdot 100 - 5 \cdot 10 + 1 - 1}{10} \to \frac{2 \cdot 100 - 50}{10}$$

$$= 15 \text{ ft/sec}$$

(b) the instantaneous velocity of the object at 3 seconds. Show all calculations and include proper units.

$$\cancel{\lim_{h \to 0} 21}$$

$$= \lim_{h \to 0} \frac{s(3+h) - s(3)}{h}$$

$$= \lim_{h \to 0} \frac{2(3+h)^2 - 5(3+h) + 1 - (2 \cdot 3^2 - 5 \cdot 3 + 1)}{h}$$

$$= \lim_{h \to 0} \frac{2(9 + 6h + h^2) - 15 - 5h + 1 - (18 - 15 + 1)}{h}$$

$$\to \lim_{h \to 0} \frac{18 + 12h + 2h^2 - 15 - 5h + 1 - 18 + 15 - 1}{h}$$

$$= \lim_{h \to 0} \frac{12h + 2h^2 - 5h}{h}$$

$$= \lim_{h \to 0} \frac{(7 + 2h)h}{h}$$

$$= \lim 7 + 2h = 7 \text{ ft/sec}$$