AP Calculus AB

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Test

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Instructions: Show work or otherwise justify your answer. No work receives no credit. Good luck and have fun-ctions! ☺

1. (4 points) Find $\frac{dy}{dx}$ if $x^3 + y^3 = 3xy$.

$$3x^2\frac{dx}{dx} + 3y^2\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y\frac{dx}{dx}$$

$$x^2 + y^2\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$x^2 - y = \frac{dy}{dx}(x - y^2)$$

$$\boxed{\frac{dy}{dx} = \frac{x^2 - y}{x - y^2}}$$

2. (4 points) If $f(x) = \begin{cases} x^2 - 2x + 3, & x \le 2 \\ -4bx + 1, & x > 2 \end{cases}$, find b in order for f(x) to be continuous.

$$2^2 - 2\cdot 2 + 3 = -4b(2) + 1$$

$$3 = -8b + 1$$

$$2 = -8b$$

$$\boxed{b = \frac{-1}{4}}$$

3. (4 points) Find $\frac{dy}{dx}$ if $y = \tan(x + y)$.

$$\frac{dy}{dx} = \left(\frac{dx}{dx} + \frac{dy}{dx}\right)\sec^2(x+y)$$

$$\frac{dy}{dx} = \left(1 + \frac{dy}{dx}\right)\sec^2(x+y)$$

$$\frac{dy}{dx} = \sec^2(x+y) + \frac{dy}{dx}\sec^2(x+y)$$

$$\frac{dy}{dx}\left(1 - \sec^2(x+y)\right) = \sec^2(x+y)$$

$$\boxed{\frac{dy}{dx} = \frac{\sec^2(x+y)}{1 - \sec^2(x+y)}}$$

4. (4 points) Find an equation of the tangent line to the graph of $x^2 + 2y^2 = 3$ at the point (1,1).

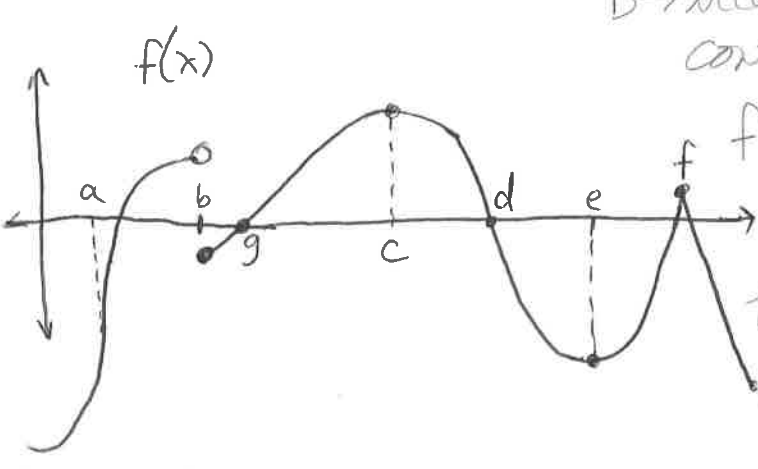
$$2x + 4y\frac{dy}{dx} = 0$$

$$4y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-x}{2y} \quad @(1,1) \rightarrow \frac{-1}{2} = m$$

$$\boxed{y - 1 = \frac{-1}{2}(x-1)}$$

5. (4 points) Use the graph of *f* below to determine all x-values at which the function is *not* differentiable. Explain your reasoning for each choice.



b → because f is not continuous @ x = b.

f because there is a corner. The derivative of f from the left is not the same as the derivative from the right

6. (8 points) A right circular cylinder is changing shape. The radius is increasing at the rate of 3 inches per second while its height is decreasing at the rate of 4 inches per second. When the radius is 2 inches and the height is 3 inches, how fast is (a) the volume ($V = \pi r^2 h$) changing and (b) the surface area ($S = 2\pi rh$) changing? Be sure to include correct units.

$$\frac{dr}{dt} = 3\,\frac{in}{sec} \qquad \frac{dh}{dt} = -4\,\frac{in}{sec} \qquad r = 2\,in \quad h = 3\,in$$

$$\frac{dV}{dt} = \pi r^2\frac{dh}{dt} + 2\pi r \cdot \frac{dr}{dt} \cdot h$$

$$\frac{dV}{dt} = \pi(2)^2 \cdot (-4) + 2\pi(2)\cdot(3)\cdot 3$$

$$\frac{dV}{dt} = -16\pi + 36\pi$$

$$\frac{dV}{dt} = 20\pi\,\frac{in^3}{sec}$$

$$S = 2\pi rh$$

$$\frac{dS}{dt} = 2\pi r\frac{dh}{dt} + 2\pi h\frac{dr}{dt}$$

$$\frac{dS}{dt} = 2\pi \cdot 2 \cdot -4 + 2\pi \cdot 3 \cdot 3$$

$$\frac{dS}{dt} = -16\pi + 18\pi$$

$$\frac{dS}{dt} = 2\pi\ in^2/sec$$

7. (4 points) An ant is walking along the curve $x^2 + xy + y^2 = 19$. If the ant is moving *down* at the rate of 3 centimeters per second, how fast is the ant moving *right or left* when the ant reaches the point (2,3)? Be sure to specify direction.

$$2x\frac{dx}{dt} + x\frac{dy}{dt} + y\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(2)\frac{dx}{dt} + 2(-3) + 3\frac{dx}{dt} + 2\cdot 3\cdot(-3) = 0$$

$$4\frac{dx}{dt} - 6 + 3\frac{dx}{dt} - 18 = 0$$

$$7\frac{dx}{dt} = 24$$

$$\frac{dx}{dt} = \frac{24}{7}\frac{cm}{sec}$$ to the right

8. (4 points) A particle moves along a vertical line with position $x(t) = \frac{8}{t^2}$. Describe its motion at *t=1*.

$x(t) = 8t^{-2}$

$x'(t) = -16t^{-3} = \frac{-16}{t^3}$

$x''(t) = 48t^{-4} = \frac{48}{t^4}$

$x'(1) = -16 \rightarrow$ moving down

$x''(1) = 48$

Since $x'(t)$ and $x''(t)$ have opposite signs, the particle is slowing down.

9. (4 points) If a ball is thrown vertically into the air so that its position above the earth at time *t* is given by the equation $s(t) = 50t - 16t^2$, find (a) its average velocity between time *t=1* and *t=2* and (b) its instantaneous velocity at *t=1*. Be sure to include correct units.

(a) avg. velocity $= \frac{s(1) - s(2)}{1-2} = \frac{50-16-(100-48)}{-1} = \frac{34-52}{-1} = 18\frac{ft}{sec}$

(b) $s'(t) = 50 - 32t$

$s'(1) = 50 - 32$

$s'(1) = 18$ ft/sec