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Chapter 7: Sampling Distributions

Section 7.2
Sample Proportions

The Practice of Statistics, 4th edition – For AP*

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Chapter 7
Sampling Distributions

- 7.1 What is a Sampling Distribution?
- **7.2 Sample Proportions**
- 7.3 Sample Means

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Section 7.2
Sample Proportions

Learning Objectives

After this section, you should be able to…

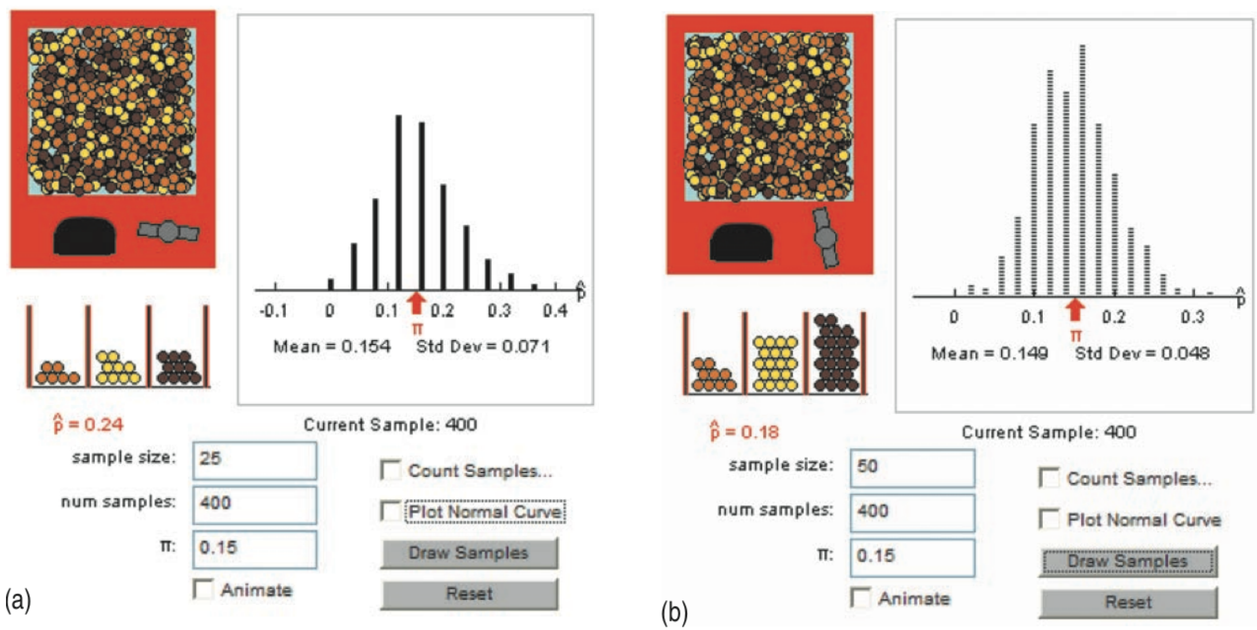
- ✓ FIND the mean and standard deviation of the sampling distribution of a sample proportion
- ✓ DETERMINE whether or not it is appropriate to use the Normal approximation to calculate probabilities involving the sample proportion
- ✓ CALCULATE probabilities involving the sample proportion
- ✓ EVALUATE a claim about a population proportion using the sampling distribution of the sample proportion

The Sampling Distribution of $\hat{p}$

How good is the statistic $\hat{p}$ as an estimate of the parameter p? The sampling distribution of $\hat{p}$ answers this question.

Consider the approximate sampling distributions generated by a simulation in which SRSs of *Reese's Pieces* are drawn from a population whose proportion of orange candies is either 0.45 or 0.15.

What do you notice about the shape, center, and spread of each?



The Sampling Distribution of $\hat{p}$

What did you notice about the shape, center, and spread of each sampling distribution?

Shape: In some cases, the sampling distribution of $\hat{p}$ can be approximated by a Normal curve. This seems to depend on both the sample size n and the population proportion p.

Center: The mean of the distribution is $\mu_{\hat{p}} = p$. This makes sense because the sample proportion $\hat{p}$ is an unbiased estimator of p.

Spread: For a specific value of p, the standard deviation $\sigma_{\hat{p}}$ gets smaller as n gets larger. The value of $\sigma_{\hat{p}}$ depends on both n and p.

There is and important connection between the sample proportion $\hat{p}$ and the number of "successes" X in the sample.

$$\hat{p} = \frac{\text{count of successes in sample}}{\text{size of sample}} = \frac{X}{n}$$

$$\frac{X}{n}$$

Sample Proportions

The Sampling Distribution of $\hat{p}$

In Chapter 6, we learned that the mean and standard deviation of a binomial random variable *X* are

$$\mu_X = np \qquad \sigma_X = \sqrt{np(1-p)}$$

Since $\hat{p} = X/n = (1/n) \cdot X$, we are just multiplying the random variable *X* by a constant (1/*n*) to get the random variable $\hat{p}$. Therefore,

$$\hat{p} = \frac{X}{n} = \frac{1}{n} \cdot X$$

$$\mu_{\hat{p}} = \frac{1}{n} \cdot np = p$$

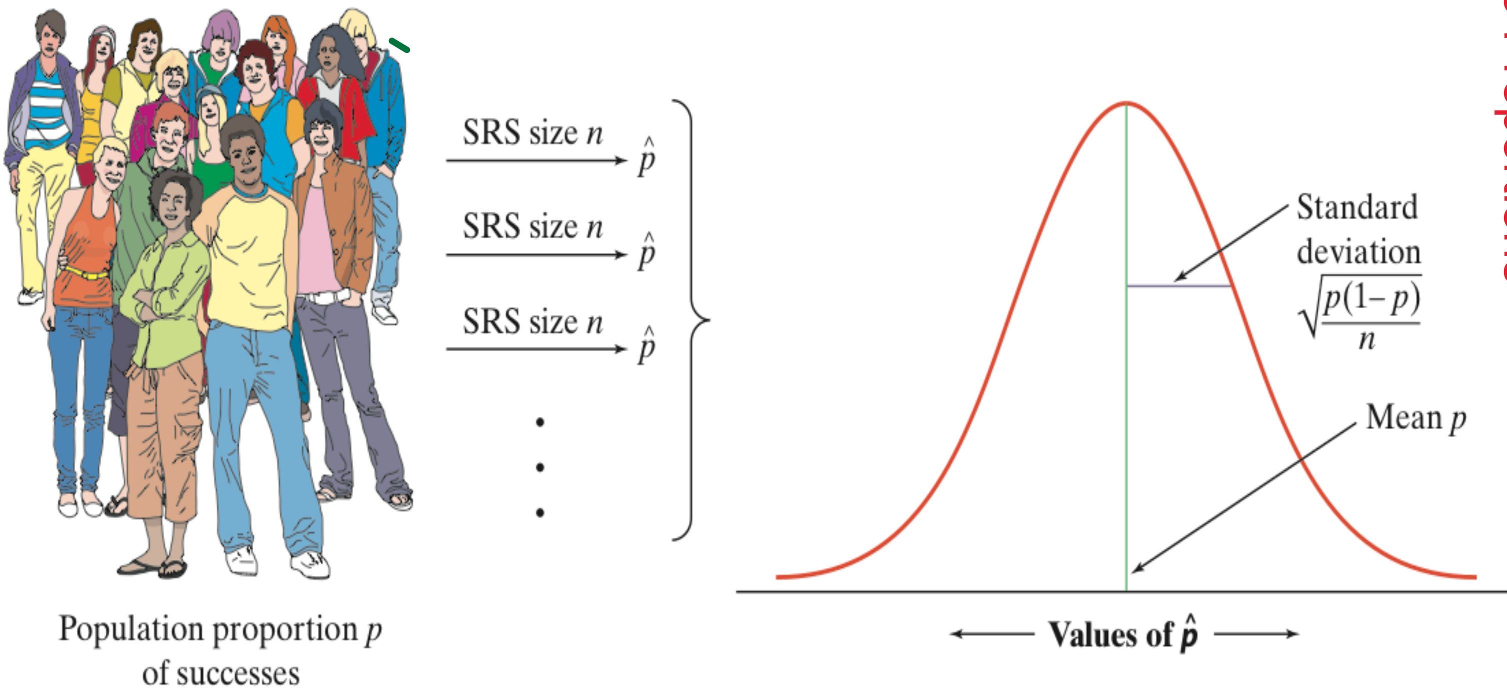
$\hat{p}$ is an unbiased estimator or *p*

$$\sigma_{\hat{p}} = \frac{1}{n}\sqrt{np(1-p)} = \sqrt{\frac{np(1-p)}{n^2}} = \sqrt{\frac{p(1-p)}{n}}$$

As sample size increases, the spread decreases.

The Sampling Distribution of $\hat{p}$

We can summarize the facts about the sampling distribution of $\hat{p}$ as follows:



Using the Normal Approximation for $\hat{p}$

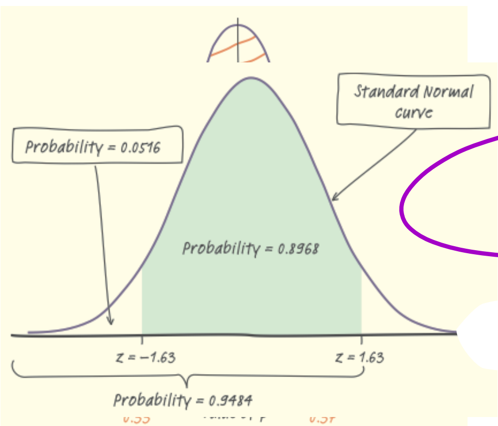
Inference about a population proportion p is based on the sampling distribution of $\hat{p}$. When the sample size is large enough for np and $n(1-p)$ to both be at least 10 (the Normal condition), the sampling distribution of $\hat{p}$ is approximately Normal.

STEP 4

A polling organization asks an SRS of 1500 first-year college students how far away their home is. Suppose that 35% of all first-year students actually attend college within 50 miles of home. What is the probability that the random sample of 1500 students will give a result within 2 percentage points of this true value?

STATE: We want to find the probability that the sample proportion falls between 0.33 and 0.37 (within 2 percentage points, or 0.02, of 0.35).

PLAN: We have an SRS of size n = 1500 drawn from a population in which the proportion p = 0.35 attend college within 50 miles of home.



DO: Since np = 1500(0.35) = 525 and $n(1-p)$ = 1500(0.65) =975 are both greater than 10, we'll standardize and then use Table A to find the desired probability.

CONCLUDE: About 90% of all SRSs of size 1500 will give a result within 2 percentage points of the truth about the population.

Section 7.2
Sample Proportions

Summary

In this section, we learned that…

- ✓ When we want information about the population proportion p of successes, we often take an SRS and use the sample proportion $\hat{p}$ to estimate the unknown parameter p. The **sampling distribution** of $\hat{p}$ describes how the statistic varies in all possible samples from the population.
- ✓ The **mean** of the sampling distribution of $\hat{p}$ is equal to the population proportion p. That is, $\hat{p}$ is an unbiased estimator of p.
- ✓ The **standard deviation** of the sampling distribution of $\hat{p}$ is $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$ for an SRS of size n. This formula can be used if the population is at least 10 times as large as the sample (the 10% condition). The standard deviation of $\hat{p}$ gets smaller as the sample size n gets larger.
- ✓ When the sample size n is larger, the sampling distribution of $\hat{p}$ is close to a Normal distribution with mean p and standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$.
- ✓ In practice, use this Normal approximation when both $np \geq 10$ and $n(1 - p) \geq 10$ (the Normal condition).

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Looking Ahead…

In the next Section…

We'll learn how to describe and use the sampling distribution of sample means.

We'll learn about

- ✓ **The sampling distribution of** $\bar{x}$
- ✓ **Sampling from a Normal population**
- ✓ **The central limit theorem**