

$$f(x) = \frac{1}{\sqrt{x}}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

Common Denominator

$$f'(x) = \lim_{h \rightarrow 0} \left(\frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \right) \left(\frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} \right)$$

Rationalize Numerator

$$f'(x) = \lim_{h \rightarrow 0} \frac{x - (x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$$

Simplify

$$f'(x) = \lim_{h \rightarrow 0} \frac{-h}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$$

$$f'(x) = \frac{-1}{\sqrt{x}\sqrt{x}(\sqrt{x} + \sqrt{x})}$$

$$f'(x) = \frac{-1}{x \cdot 2\sqrt{x}} \text{ OR } -\frac{1}{2} x^{-3/2}$$

Evaluate when $h=0$