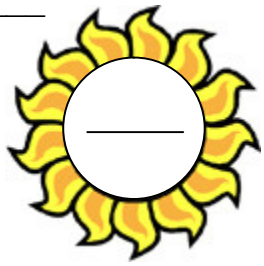
BC Calculus: NAME _____
Summer Assignment: 2011 DATE _____ PER _____



Complete each of the following. Show all work to receive credit.

I. Algebra Review (10 points)

_____ 1) Factor and simplify $x(x-1)^{-\frac{1}{2}} + 2(x-1)^{\frac{1}{2}}$.
Express the answer as a fraction without negative exponents.

_____ 2) Express $\dfrac{\dfrac{1}{y-k} - \dfrac{1}{y}}{k}$ as a simple fraction.

_____ 3) Multiply $\left(x^{\frac{3}{2}} + \dfrac{2}{\sqrt{3}}\right)^2$.

_____ 4) Solve $x(x-2)^{-\frac{1}{2}} + 3(x-2)^{\frac{1}{2}} = 10.$ for x.

_____ 5) Solve: $|x+5| < 3$.

_____ 6) Write the equation of the line passing through the point $(3,-1)$ with slope $\dfrac{5}{2}$.

_____ 7) Solve $xz + y = 1 + z$ for z.

_____ 8) Write the equation of the circle in standard form and give the center and radius. $2x^2 + 2y^2 + 4x - 12y + 11 = 0$

Center: _____

Radius: _____

_____ 9) Solve $2(x - 5)^{-1} + \frac{1}{x} = 0$ for x.

_____ 10) Find the domain of f. $f(x) = \sqrt{2x + 3}$

II. Exponential and Logarithmic Functions (10 points)

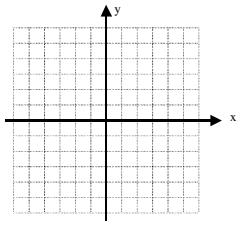
_____ 1) Solve $\ln\left(e^{7x}\right) = 15$ for x.

_____ 2) Solve $\frac{e^{x+5}}{e^5} = 3$ for x.

_____ 3) Solve $\left(e^3\right)^{2x} = e^3 e^{2x}$ for x.

_____ 4) Solve $e^{\left[2\ln x - \ln\left(x^2 + x - 3\right)\right]} = 1$ for x.

_____ 5) Solve $3^{2x} - 2 \cdot 3^{(x+5)} + 3^{10} = 0$ for x. (**HINT:** Factor!)



6) Sketch the graph of the function $f(x) = e^x$.

_____ 7) Find the x-intercept for the graph of the function $f(x) = \ln x + 2$.

_____ 8) Use the properties of logarithms to expand the expression $\ln \dfrac{(4x^5 - x - 1)\sqrt{x - 7}}{(x^2 + 1)^3}$.

_____ 9) Solve $\ln x - \ln(x + 1) = 1$ for x.

_____ 10) Find the domain of the function $f(x) = \ln(3x + 2)$.

III. Trigonometry (Remember to use trig identities and to draw right triangles!) (10 points)

_____ 1) If $\csc\theta = \dfrac{13}{5}$ and θ is in the second quadrant, find $\sec\theta$.

_____ 2) Find all θ in the interval $[0,2\pi)$ that satisfy the equation $\sin 2\theta = 0$.

_____ 3) Simplify the expression $\sqrt{x^2 + 4}$ in terms of θ when $x = 2\tan\theta$.

_____ 4) Simplify $\dfrac{\cot\theta}{\csc\theta}$.

_____ 5) Find $\sin 2A$ if $\sin A = \dfrac{1}{4}$ and $0 \le A \le \dfrac{\pi}{2}$.

_____ 6) Find all θ in the interval $[0,2\pi)$ that satisfy the equation $2\cos\theta\tan\theta + \tan\theta = 0$.

_____ 7) If $\cos 2\theta = \dfrac{1}{3}$ and $0 \le \theta \le \pi$, find $\cos\theta$.

_____ 8) Rewrite the given equation in polar form using the substitutions $x = r\cos\theta$ and $y = r\sin\theta$. Solve for r.
$x^2 + y^2 + 3x = 0$

_____ 9) Write the given expression, $\tan\left(\arccos\dfrac{x}{3}\right)$, in algebraic form.

_____ 10) Compute $\arcsin\left(\frac{-1}{2}\right)$.

IV. Polar Graphs (7 Points)

1) Change each set of rectangular coordinates to polar:

_____(3, 3)

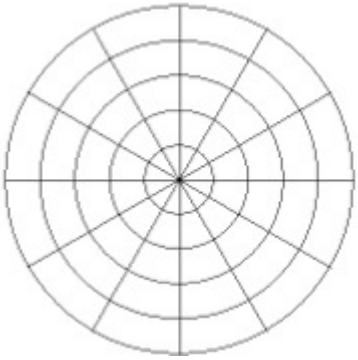
_____(0, 5)

_____$(-\frac{\sqrt{3}}{2}, \frac{1}{2})$

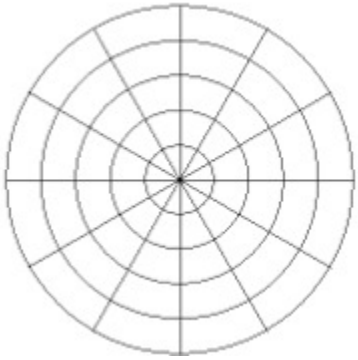
_____$(\sqrt{3}, -3)$

2) Sketch the graph:

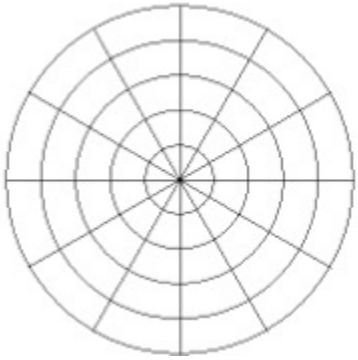
a) $r = 3$



b) $r = 1 - \cos\theta$



c) $r = \sin 3\theta$



IV. Differentiation (#1-9 Find the derivative and simplify the answer.) (10 points)

_____ 1) $\frac{d}{dx}\left(xe^{x}\right)$

_____ 2) $\frac{d}{dx}\left(\frac{x}{x+1}\right)$

_____ 3) $\frac{d}{dx}\sqrt{2x-1}$

_____ 4) $\frac{d}{dx}\left[\sin^2(2x)\right]$

_____ 5) $\frac{d}{dx}\left[\arcsin\frac{x}{2}\right]$

_____ 6) $\frac{d}{dx}e^{(x^2)}$

_____ 7) $\frac{d}{dx}\ln\left(\frac{x}{x+1}\right)$

_____ 8) $\frac{d}{dx}\left[x^2\sqrt{2x+1}\right]$

_____ 9) $x^4 - \frac{5}{x} + \sqrt{x} - x^e - \pi^3$

_____ 10) Find $\frac{dy}{dx}$: $y^3 + xy = 5$

V. Integration (#1-10 Evaluate each integral.) (10 points)

_____ 1) $\int \sqrt{2x - 1}\, dx$

_____ 2) $\int_1^2 (2x - 1)\, dx$

_____ 3) $\int \frac{1}{2x - 1} dx$

_____ 4) $\int \frac{x}{2x - 1} dx$

_____ 5) $\int \frac{3}{\sqrt{1 - x^2}} dx$

_____ 6) $\int \sin(2x - 1)\, dx$

_____ 7) $\int e^{(2x\ -\ 1)}\,dx$

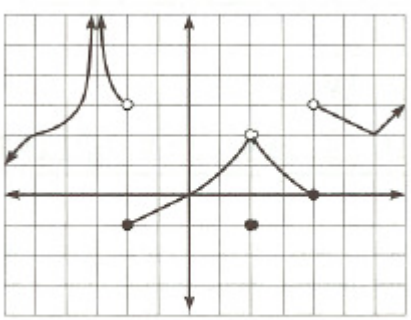
_____ 8) $\int \frac{1}{4\ +\ 4x^2}\,dx$

_____ 9) $\int \sec^2(2x\ -\ 1)\,dx$

_____ 10) $\int \frac{1}{(2x\ -\ 1)^2}\,dx$

VI. Applications of Calculus **(If multiple choice question, write the letter of the BEST answer.)**

1) If $d(x)$ is defined by the graph below, answer the following questions.


Graph of $d(x)$

_____ a) Find $\lim_{x \to 0} d(x)$. (1 point)

_____ b) Find $\lim_{x \to -2^+} d(x)$. (1 point)

_____ c) Find $\lim_{x \to -2^-} d(x)$. (1 point)

_____ d) Find $\lim_{x \to 2} d(x)$. (1 point)

_____ e) Find $\lim_{x \to 4^-} d(x)$. (1 point)

_____ f) List all x values where d is discontinuous. (1 point)

g) Which of your answers to part (f) represents removable discontinuities, and ***why***? (2 points)

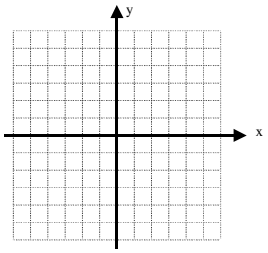
_____ h) Which of your answers to part (f) represents jump discontinuities? (1 point)

_____ i) What value(s) of β make the following statement true? (1 point)

$$\lim_{x \to \beta} d(x) = \infty$$

_____ 2) Find the limit (if one exists) of $\lim_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - 1}$. (2 points)

3) a) Sketch the graph of $f(x) = \begin{cases} x^2 + 4x + 2, & x < -2 \\ 1 - 4x - x^2, & x \geq -2 \end{cases}$. (1 point)



b) Is f continuous at $x = -2$? (Use the three parts of the definition of continuity to explain.) (3 points)

c) Is f differentiable at $x = -2$? Explain. (2 points)

_____ 4) Which of the following is $\lim_{h \to 0} \dfrac{\sin(x + h) - \sin(x)}{h}$? (2 points)

a) 0 b) 1 c) $\sin(x)$ d) $\cos(x)$ e) It is nonexistent.

_____ 5) Find the maximum value of $f(x) = 2x^3 - 9x^2 + 12x - 1$ on [-1,2]. (2 points)

a) 0 b) 1 c) 2 d) 3 e) 4

_____ 6) What are all values of x for which the graph of $y = x^3 - 6x^2$ is concave downward? (2 pts)

a) $0 < x < 4$ b) $x > 2$ c) $x < 2$

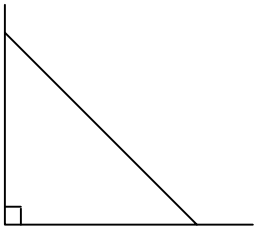
d) $x < 0$ e) $x > 4$

7) If the position (in feet) of a particle moving horizontally along the x-axis is given by the equation $s(t) = t^3 - 10t^2 + 25t, \quad t > 0$ seconds.

a) At what time is the particle **not** moving and ***why***? (3 points)

_____ b) On what interval of time is the particle moving to the left? (2 points)

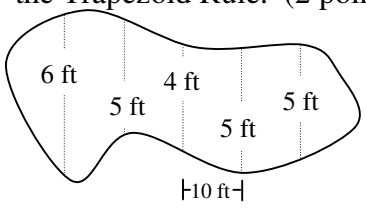
_____ 8) A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. How fast is the top of the ladder moving down the wall when the base of the ladder is 24 feet from the wall? (2 points)



_____ 9) Air is being pumped into a spherical balloon at the rate of $5 \frac{\text{in}^3}{\text{min}}$. Find the rate of change of the radius when the radius is 2 inches.

NOTE: $V_{sphere} = \frac{4}{3}\pi r^3$ (2 points)

_____ 10) To estimate the surface area of a pond, a surveyor takes several measurements, as shown in the figure. Estimate the surface area using the Trapezoid Rule. (2 points)



_____ 11) If $\int_0^4 \left(x^2 - 6x + 9\right) dx$ is approximated by 4 ***inscribed*** rectangles of equal width on the x-axis, then the approximation is which of the following? (2 points)

a) 14 b) 10 c) 6 d) 5 e) 4

_____ 12) A particle moves on the x-axis so that at any time t its velocity $v(t) = \sin 2t$ is subject to the condition $x(0) = 0$ where $x(t)$ is the position function. Which of the following is an expression for $x(t)$? (2 points)

a) $\cos 2t + \frac{1}{2}$

b) $-\frac{1}{2}\sin 2t + \frac{1}{2}$

c) $-\frac{1}{2}\cos 2t$

d) $-\frac{1}{2}\cos 2t + \frac{1}{2}$

e) $-\frac{1}{2}\cos 2t - \frac{1}{2}$

_____ 13) A particle moves along a straight line so that its velocity is given by $v(t) = t^2$. How far does the particle travel between $t = 1$ to $t = 3$? (2 points)

a) $\frac{1}{3}$ b) 8 c) $\frac{26}{3}$ d) 26 e) 27

_____ 14) Find the area of the region between the graph of $y = 3x^2 + 2x$ and the x-axis from $x = 1$ to $x = 3$. (2 points)

a) 36 b) 34 c) 31 d) 26 e) 12