AP Calculus AB Name:_____

Area between curves, Average Value Wednesday, March 6, 2013

Instructions: **You MAY use a calculator for this part.** Be sure to set up the problems and to show work. Round final answers to at least three decimal places. Circle your answer choice on the multiple choice problems which are worth one point each.

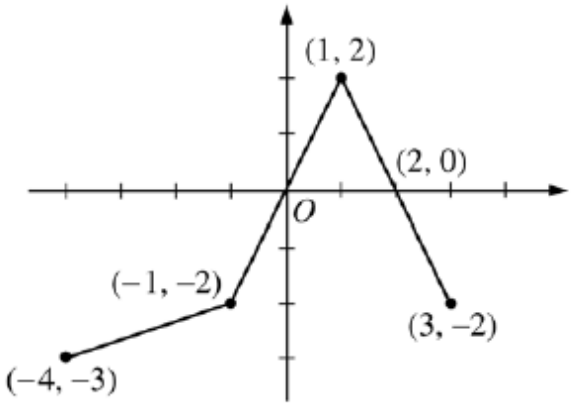
(4 points) 1. Let *f* and *g* be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{\frac{x}{2}}$. Let *R* be the region in the first quadrant enclosed by the graphs of *f* and *g*. Sketch a graph of *R* and find its area.

(3 points) 2.

The graph of the function f above consists of three line segments.

(a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$.

For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.



Graph of f

2. A particle moves along the *x*-axis so that its velocity *v* at time *t*, for $0 \leq t \leq 5$, is given by

$$v(t) = \ln(t^2 - 3t + 3).$$

The particle is at position *x=8* at time *t=0*.

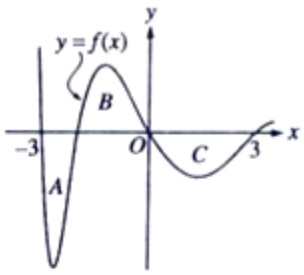
(a) 1 point: Find the acceleration of the particle at time *t=4*.

(b) 3 points: Find all time *t* in the open interval 0<*t*<5 at which the particle changes direction. During which time intervals, for $0 \leq t \leq 5$, does the particle travel to the left?

(c) 3 points: Find the position of the particle at time *t=2*.

(d) 2 points: Find the average speed of the particle over the interval $0 \leq t \leq 2$.

4.



The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 3, what is the value of $\int_{-3}^{3}(f(x) + cosx)dx$?

A) -5.718
B) -2.718
C) .282
D) 3.282
E) 6.282

5.

Let g be the function given by $g(x) = \int_0^x \cos(et^2 - \pi)\, dt$ for $-1 \leq x \leq 1$. On which of the following intervals is g decreasing?

A) $-1 \leq x \leq -.760$
B) $-.760 \leq x \leq .760$
C) $.760 \leq x \leq 1$
D) $-1 \leq x \leq 0$
E) $0 \leq x \leq 1$

6.

The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t - te^t + 3e$. What is the average velocity of the particle from time $t = -3$ to time $t = 2$.

A) 8.105
B) 37.525
C) 40.525
D) 42.525
E) 202.626

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Instructions: **You MAY NOT use a calculator for this part.** Justify your answers. Circle your answer choice on the multiple choice problems which are worth one point each. Good luck and have fun-ctions!

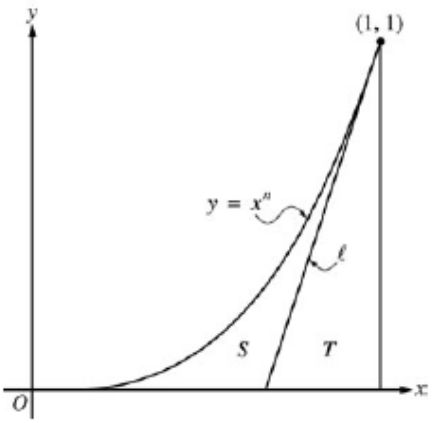
1. (9 points: 2,3,4)

Let ℓ be the line tangent to the graph of $y = x^n$ at the point $(1, 1)$, where $n > 1$, as shown above.

(a) Find $\int_0^1 x^n \, dx$ in terms of n.

(b) Let T be the triangular region bounded by ℓ, the x-axis, and the line $x = 1$. Show that the area of T is $\frac{1}{2n}$.

(c) Let S be the region bounded by the graph of $y = x^n$, the line ℓ, and the x-axis. Express the area of S in terms of n and determine the value of n that maximizes the area of S.



2. $\int_0^{\pi/4} \cos(x)\,dx =$

A) $-\frac{\sqrt{2}}{2}$

B) $\frac{\sqrt{2}}{2}$

C) $-\frac{\sqrt{2}}{2} - 1$

D) $-\frac{\sqrt{2}}{2} + 1$

E) $\frac{\sqrt{2}}{2} - 1$

3. $\lim_{x\to\infty} \frac{x^3-3x^2+4x-5}{5x^3-3x^2+2x-3} =$

A) 5

B) 1

C) $\frac{1}{5}$

D) 0

E) $\frac{5}{3}$

4. (8 points) Find the area between $y = 16 - 4x - x^2$ and $y = 4 - 3x$. Sketch a graph and be sure to label the points of intersection. (Show work to find the intersections.) Write the integral that represents the area. Evaluate the integral. You do not have to simplify your final answer.

5. (4 points) Determine the average value of $f(t) = t^2 - 5t + 6\cos(\pi t)$ on $\left[-1, \frac{5}{2}\right]$. You do not have to simplify your final answer.

6. (6 points) Determine the number *c* that satisfies the Mean Value Theorem for Integrals for the function $f(x) = x^2 + 3x + 2$ on the interval [1,4].