AP Calculus AB

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Test

51

Wednesday, September 5, 2012

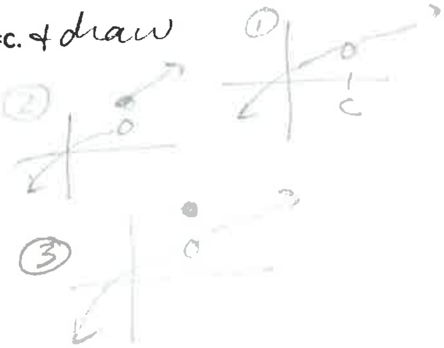
Instructions: Be sure to follow the directions for each section. SHOW WORK! Good luck and have fun-ctions! Calculators *are* permitted on this test.

(6 points) 1. List the three requirements for a function to be continuous at x=c. + draw

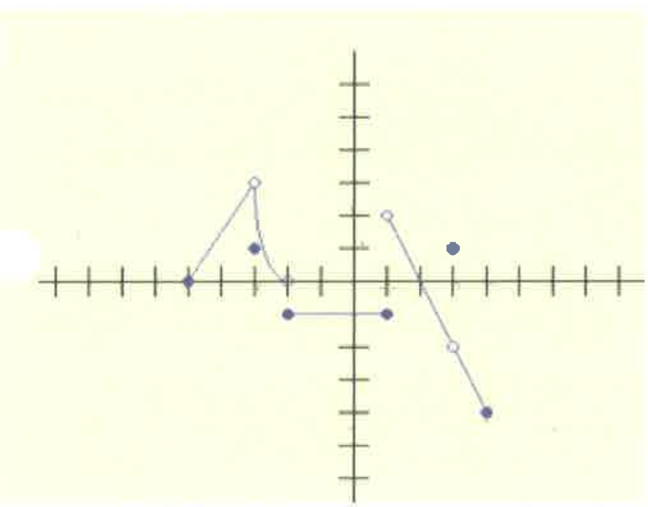
① $f(c)$ exists

② $\lim_{x\to c} f(x)$ exists

③ $\lim_{x\to c} f(x) = f(c)$



2. (1 point each) Using the graph, evaluate each. (Scale is by 1's)



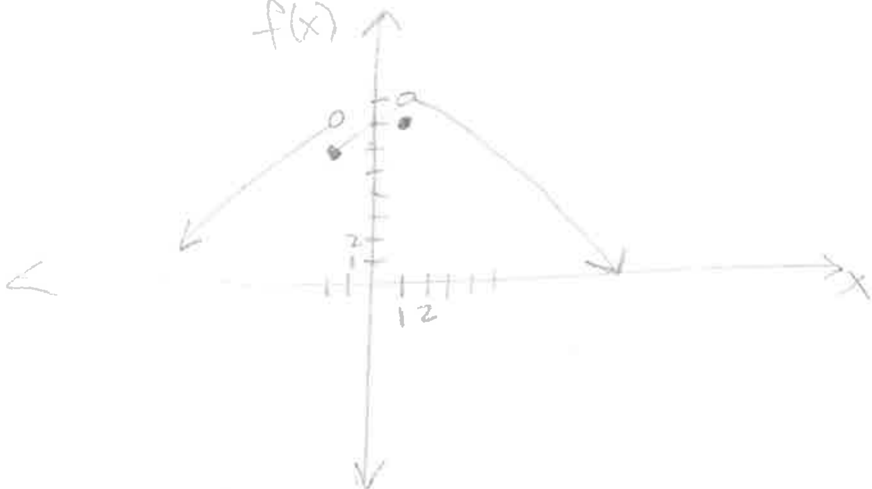
a. $\lim_{x\to 3} g(x)$ −2	b. $\lim_{x\to 1} g(x)$ dne
c. $\lim_{x\to 0} g(x)$ −1	d. $\lim_{x\to -2^-} g(x)$ 0
e. $\lim_{x\to -3} g(x)$ 3	f. $\lim_{x\to 4} g(x)$ dne
g. $\lim_{x\to 1^+} g(x)$ 2	h. $\lim_{x\to 2} g(x)$ 0
i. $\lim_{x\to 1^-} g(x)$ −1	j. $\lim_{x\to -2^+} g(x)$ −1
k. $f(-3)$ = 1	l. $f(-2)$ −1
m. $f(0)$ −1	n. $f(1)$ −1
o. $f(3)$ 1	p. $f(4)$ −4

22

3. Sketch an example of a *function* with the given attributes.

5pts.

- f(1)=7
- f(-2)=6
- The limit as x approaches -2 of f(x) does not exist; however, both one-sided limits exist.
- The limit as x approaches 1 of f(x) is equal to 8.
- The function is decreasing on (1,∞)



4. (6 points) Consider the following function:

$$f(x) = \begin{cases} a + bx, & if\ x > 2 \\ 3, & if\ x = 2 \\ b - ax^2, & if\ x < 2 \end{cases}$$

Determine the values of constants *a* and *b* so that $\lim_{x \to 2} f(x)$ exists and is equal to *f(2)*.

$f(2)=3$

$a+b(2)=3 \longrightarrow a+2b=3$

$b-a(2)^2=3 \longrightarrow {}^{-2}(-4a+b=3)$

$a+2b=3$

$8a+{}^-2b=-6$

$9a=-3$

$a=\frac{-1}{3}$

$\frac{-1}{3}+2b=3$

$2b=\frac{\cancel{8}10}{3}$

$b=\frac{5}{3}$

11

5. (8 points) Find the ***equation*** of the tangent line to $f(x) = 14 - 3x^2$ at *x=-2*. Show all calculations and include a graph.

★ specify method ★

$X_0 = -2$ $\quad f(-2) = 14 - 3 \cdot 4$ 2

$$m_{tan} = \frac{f(x_0 + h) - f(x_0)}{h}$$

$$m_{tan} = \frac{f(h-2) - 2}{h}$$

$$m_{tan} = \frac{14 - 3(h^2 - 4h + 4) - 2}{h}$$

$$m_{tan} = \frac{14 - 3h^2 + 12h - 12 - 2}{h}$$

$$m_{tan} = \frac{h(-3h + 12)}{h}$$

$$m_{tan} = -3h + 12$$

As $h \to 0$, $m_{tan} \to 12$

$y - 2 = 12(x + 2)$

OR

$y = 12x + 26$

6. (4 points) Let $f(x) = x^2 + 2x$. What is the average rate of change of *f(x)* on the interval [1,3]?

$$\text{avg R of C} = \frac{f(3) - f(1)}{3 - 1}$$

$$= \frac{9 + 6 - (1 + 2)}{2}$$

$$= \frac{12}{2} = 6$$

7. (2 points) If the limit as x approaches 2 is 4, what does that tell you about f(2)? Explain.

Nothing. The limit is the intended height of a function, not necessarily the height.

8. (2 points) If f(2) = 4, what does that tell you about the limit of f(x) as x approaches 2? Explain.

Nothing. The height of a function at a given x-value has nothing to do with its limit (intended height)

9. (2 points) Write, in words, how to say this: $\lim_{x \to 3} f(x) = 4$

The limit of f(x) as x approaches 3 is 4.

18