AP Calculus AB

Name: Mrs. Rivera

Limits & Continuity Assessment

Instructions: Show work or provide an explanation. Calculators are not permitted.
Good luck and have fun-ctions. May the limit of your grade be positive infinity!

$\frac{0}{0}$ 1) Find $\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}$. Show all work. (3 points)

$$\lim_{x\to 0}\left(\frac{\sqrt{x+2}-\sqrt{2}}{x}\right)\left(\frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}\right)$$

$$\lim_{x\to 0}\frac{x+2-2}{x(\sqrt{x+2}+\sqrt{2})}$$

$$\lim_{x\to 0}\frac{x}{x(\sqrt{x+2}+\sqrt{2})}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{x+2}+\sqrt{2}}$$

$$=\frac{1}{2\sqrt{2}}$$

2) Let $F(x)=\begin{cases}\dfrac{x^2-5x-6}{x-6}, & x\neq 6\\ 3k+2, & x=6\end{cases}$

$\dfrac{(x-6)(x+1)}{}$

a) Find $\lim_{x\to 6} F(x)$. Show all proper steps. (1 points)

$$\lim_{x\to 6^-}\frac{(x-6)(x+1)}{x-6}=7 \qquad \lim_{x\to 6^+} x+1=7$$

$$\lim_{x\to 6} F(x)=7$$

b) Find the value of k such that $\lim_{x\to 6} F(x)=F(6)$. Show all work. (2 points)

$$3k+2=7$$
$$3k=5$$
$$k=5/3$$

3. (3 points)

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

Show that f is continuous at $x = 0$.

① $\lim_{x \to 0^-} 1 - 2\sin x = 1 \rightarrow \lim_{x \to 0^+} e^{-4x} = 1 \therefore \lim_{x \to 0} f(x) = 1$

② $f(0) = 1 - 2\sin 0 = 1$

③ Since $\lim_{x \to 0} f(x) = f(0)$, $f(x)$ is continuous at $x = 0$.

4. (3 points) Mrs. Rivero drops her calculus book off of the top of a 220-foot building. The position function of the book is: $s(t) = -16t^2 + 220$. Using the velocity function below, find the velocity of the book when t = 1 second. Be sure to include units and to show all steps.

$\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$

5. (2 points) Use the Intermediate Value Theorem to show that $f(x)=-x^2 + 2x + 4$ has at least one zero on [1,4].

Since $f(x)$ is a polynomial, it is continuous everywhere.

$f(1) = -1+2+4 = 5$

$f(4) = -16+8+4 = -4$

Since $f(1) > 0$ and $f(4) < 0$, $f(x)$ must cross the x-axis (have a zero) on $[1,4]$.