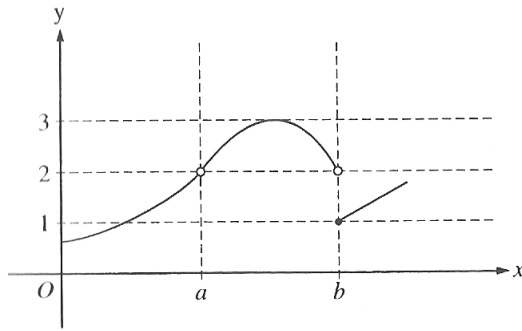Blizzard Bag #2

Find the following algebraically.

1. $\lim\limits_{x \to 3} \dfrac{x-3}{x^2 - 2x - 3}$ is

   (A) 0 　　(B) 1 　　(C) $\dfrac{1}{4}$ 　　(D) ∞ 　(E) none of these

2. $\lim\limits_{x \to 0} \dfrac{|x|}{x}$ is

   (A) 0 　　(B) nonexistent 　(C) 1 　(D) -1 　　(E) none of these

3. $\lim\limits_{x \to 7} \dfrac{x-7}{\sqrt{x}-7}$ is

   (A) $2\sqrt{7}$ 　(B) $\sqrt{7}$ 　(C) 0 　　(D) $-2\sqrt{7}$ 　　(E) nonexistent

Find the following WITHOUT the use of a calculator.

4. $\lim\limits_{x \to 1} \dfrac{x}{\ln x}$ is

   (A) 0 　　(B) $\dfrac{1}{e}$ 　　(C) 1 　　(D) e 　　(E) nonexistent

5. If $a \neq 0$, then $\lim\limits_{x \to a} \dfrac{x^2 - a^2}{x^4 - a^4}$ is

   (A) $\dfrac{1}{a^2}$ 　(B) $\dfrac{1}{2a^2}$ 　(C) $\dfrac{1}{6a^2}$ 　(D) 0 　　(E) nonexistent

6. $\lim\limits_{x \to \infty} \dfrac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$

   (A) 4 　　(B) 1 　　(C) $\dfrac{1}{4}$ 　　(D) 0 　　(E) -1

7. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to b} f(x)$

(B) $\lim\limits_{x \to a} f(x) = 2$

(C) $\lim\limits_{x \to b} f(x) = 2$

(D) $\lim\limits_{x \to b} f(x) = 1$

(E) $\lim\limits_{x \to a} f(x)$ does not exist.

8. Let $f(x) = 4 - 3x$. Which of the following is equal to $f'(-1)$?

(A) -7   (B) 7   (C) -3   (D) 3   (E) nonexistent

9. Which of the following is true about the graph of $f(x) = x^{\frac{4}{5}}$ at $x = 0$?

    (A) It has a corner.
    (B) It has a cusp.
    (C) It has a vertical tangent.
    (D) It has a discontinuity.
    (E) $f(0)$ does not exist.

10. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

       I. f is continuous at $x = 0$.
      II. f is differentiable at $x = 0$.
      III. f has an absolute minimum at $x = 0$.

(A) I only   (B) II only   (C) III only

(D) I and III only   (E) II and III only

Free Response...No calculator

1. Let $f(x) = \begin{cases} x^2 - a^2 x & \text{if } x < 2 \\ 4 - 2x^2 & \text{if } x \geq 2 \end{cases}$.

  (a) Find $\lim\limits_{x \to 2^-} f(x)$.

  (b) Find $\lim\limits_{x \to 2^+} f(x)$.

  (c) Find all values of a that make
      f continuous at 2. Justify your answer.

2. Let $f(x) = 2x - x^2$.

  (a) Find $f(4)$

  (b) Find $f(4 + h)$

  (c) Find $\dfrac{f(4+h) - f(4)}{h}$

  (d) Find the instantaneous rate of change
      of f at $x = 4$.