AP Calculus AB
Wednesday, September 25, 2013

Test Friday over derivatives!
This week, we will learn the product rule, quotient rule, and chain rule.
You need to be here for ETEH everyday! :) Donald is 2 for 2!!!

Actual AP Problems!

19. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x,y) on the graph of f have the property that the line tangent to f at (x,y) has slope $\frac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\frac{1}{2},\frac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4,2)$

(D) $(0,0)$ and $\left(4,\frac{2}{3}\right)$

(E) There are no such points.

$$f'(x) = \frac{(x+2)(1)-(x)(1)}{(x+2)^2}$$

$$f'(x) = \frac{x+2-x}{(x+2)^2}$$

$$f'(x) = \frac{2}{(x+2)^2}$$

$$\frac{2}{(x+2)^2} = \frac{1}{2}$$

$$(x+2)^2 = 4$$

$$f(x) = \frac{x}{x+2}$$

$$x+2 = \pm 2$$

$$x = -2 \pm 2 \to \left.\begin{matrix} -4 \\ 0 \end{matrix}\right\}$$

need matching y-values

$(0,0)$ $(-4,2)$

$f(x) = (2x-1)^4$

Up to now, we would have to expand this in order to find $f'(x)$.

This $f(x)$ can be thought of as $f(x) = h(g(x))$ where $h(x) = x^4$ and $g(x) = 2x-1$.

Ex. $k(x) = e^x$, $l(x) = \frac{1}{x-2}$

$k(l(x)) = e^{\frac{1}{x-2}}$

$l(k(x)) = \frac{1}{e^x - 2}$

In order to take the derivative of compositions of functions, we will use the CHAIN RULE.

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

Ex. $f(x) = (2x-1)^4$

$f'(x) = 4(2x-1)^3 \cdot 2$

$f'(x) = 8(2x-1)^3$

Result:

$16x^4 - 32x^3 + 24x^2 - 8x + 1 = f(x)$

$f'(x) = 64x^3 - 96x^2 + 48x - 8$

w/ Chain Rule

$f'(x) = 8(2x-1)^3$

same (check by expanding)

Ex. $g(x) = (3-5x)^{1/5}$

$g'(x) = \frac{1}{5}(3-5x)^{-4/5} \cdot (-5)$

$g'(x) = -1(3-5x)^{-4/5}$

$= \frac{-1}{\sqrt[5]{(3-5x)^4}}$

Exs. find derivative:

① $y = (x^2-4x+9)^3$

$y' = 3(x^2-4x+9)^2 \cdot (2x-4)$

② $y = (\sqrt{x}-2)^4$

$y' = 4(\sqrt{x}-2)^3 \cdot \frac{1}{2}x^{-1/2}$

$y' = \frac{2(\sqrt{x}-2)^3}{\sqrt{x}}$

③ $y = \boxed{(x^2-5x+1)^2} \cdot \boxed{(3x-4)}$

$y' = (x^2-5x+1)^2 \cdot (3) + (3x-4)(2(x^2-5x+1)(2x-5))$

3. Find f'(x) if $f(x) = \sqrt{x^2-4x+3}$

4. Find g'(x) if $g(x) = \frac{1}{5x-3}$

$f(x) = (x^2-4x+3)^{1/2}$

$f'(x) = \frac{1}{2}(x^2-4x+3)^{-1/2}(2x-4)$

$f'(x) = \frac{x-2}{\sqrt{x^2-4x+3}}$

$g(x) = \frac{1}{5x-3}$

$g'(x) = \frac{(5x-3)(0) - 1 \cdot 5}{(5x-3)^2}$

$g'(x) = \frac{-5}{(5x-3)^2}$

Look @ $g(x) = \frac{1}{5x-3}$ as chain Rule

$g(x) = (5x-3)^{-1}$

$g'(x) = -1(5x-3)^{-2} \cdot 5$

$g'(x) = -5(5x-3)^{-2}$

$g'(x) = \frac{-5}{(5x-3)^2}$

12) $y = \left(\frac{2x-1}{2x+1}\right)^5$

$y' = 5\left(\frac{2x-1}{2x+1}\right)^4 \cdot \frac{d}{dx}\left(\frac{2x-1}{2x+1}\right)$ → Derivative of $\frac{2x-1}{2x+1}$

$y' = 5\left(\frac{2x-1}{2x+1}\right)^4 \cdot \frac{(2x+1)(2) - (2x-1)(2)}{(2x+1)^2}$

$y' = 5\left(\frac{2x-1}{2x+1}\right)^4 \cdot \frac{4x+2-(4x-2)}{(2x+1)^2}$

$y' = 5\left(\frac{2x-1}{2x+1}\right)^4 \cdot \frac{4}{(2x+1)^2}$

$y' = \frac{20(2x-1)^4}{(2x+1)^6}$

Given that $f(2) = -3, f'(2) = 6, g(2) = 3, g'(2) = -2, f'(3) = 4$, find the derivatives of the following at $x = 2$.

16) $f(x) \cdot g(x)$ 17) $\frac{f(x)}{g(x)}$ 18) $[f(x)]^3$ 19) $f(g(x))$

⑱ $3 \cdot [f(x)]^2 \cdot f'(x)$

$3 \cdot [f(2)]^2 \cdot f'(2)$

$3 \cdot [-3]^2 \cdot 6$

$3 \cdot 9 \cdot 6$

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$f'(g(x)) \cdot g'(x)$

@ $x=2$

$f'(g(2)) \cdot g'(2)$

$f'(3) \cdot g'(2)$

$4 \cdot -2$

$\boxed{8}$