AP Calculus AB
Thursday, September 20, 2012

Quiz tomorrow on basic derivative rules
Today's Essential Question: What is the product rule?

$$y = \frac{x+2}{\sqrt{x}}$$

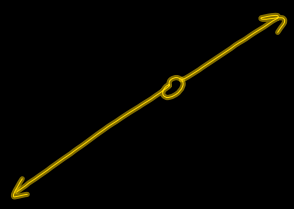
$$y = \frac{x^1}{\sqrt{x}} + \frac{2}{\sqrt{x}}$$

$$y = x^{1/2} + 2x^{-1/2}$$

$$y' = \frac{1}{2}x^{-1/2} + -1\,x^{-3/2}$$

$$y' = \frac{1}{2\sqrt{x}} - \frac{1}{\sqrt{x^3}}$$

$$f(x) = \frac{x^2-9}{x+3}$$



$$f(x) = x-3$$
$$f'(x) = 1$$

$$y = \sqrt{3}x + \frac{x}{\pi+1}$$

$$y = \sqrt{3}x + \frac{1}{\pi+1}x$$

$$y' = \sqrt{3} + \frac{1}{\pi+1}$$

$$f(x) = \frac{\pi^e \cdot e^{\sqrt{7}}}{\sqrt{\pi} + \frac{e}{1.8}}$$

$f'(x) = 0$ b/c $f(x)$ is a constant.

Product Rule >

If $f(x)=x$ $\quad g(x)=x^2$

$f'(x)=1$ $\quad g'(x)=2x$

$(f\cdot g)(x)=x^3$

$f'(x)g'(x) \neq (f\cdot g)'(x)=3x^2$

Leibniz

Discovering the Product Rule

Given a rectangle with initial length 5 cm and initial width 9 cm. Determine the rate of change of the area given that the length is increasing at 3 cm/sec and the width is increasing at 2 cm/sec.

Write expressions for the width and length of the rectangle at time t using the format $w(t)$ and $l(t)$. Find $w'(t)$ and $l'(t)$. Then, create an expression for the area (use $A(t)$) of the rectangle at time t using the definitions you wrote for the width and length. Then, find the expression for the rate of change of the area.

$w(t)=2t+9$

$l(t)=3t+5$

$l'(t)=3$

$w'(t)=2$

$A(t)=(2t+9)(3t+5)$

$A'(t)=$

$A(t)=6t^2+37t+45$

Create a table to investigate the relationship between $w(t)$, $l(t)$, $w'(t)$, $l'(t)$ and $A'(t)$.

w(t)	l(t)	w'(t)	l'(t)	A'(t)
9	5	2	3	37
11	8	2	3	49
13	11	2	3	61
15	14	2	3	73

Write your hypothesis for the relationship using function notation.

$A(t)=w(t)\cdot l(t)$

$A'(t)=w(t)\cdot l'(t)+l(t)\cdot w'(t)$

$A'(t)=(2t+9)\cdot 3+(5+3t)\cdot 2$

$A'(t)=6t+27+10+6t$

$A'(t)=12t+37$

Original problem: $f(x)=x$ $\quad g(x)=x^2$

$f'(x)=1$ $\quad g'(x)=2x$

$(f\cdot g)'(x)=f(x)\cdot g'(x)+g(x)\cdot f'(x)$

$=x\cdot 2x+x^2\cdot 1$

$=2x^2+x^2$

$=3x^2$ ☺

$y=(2x+1)^2$

expand & use power rule

$y=4x^2+4x+1$

$y'=8x+4$

OR

use product rule:

$y=(2x+1)(2x+1)$

$y'=(2x+1)(2)+(2x+1)(2)$

$y'=4x+2+4x+2$

$y'=8x+4$ ☺

Create a table to investigate the relationship between w(t), l(t), w'(t), l'(t) and A'(t).

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