AP Calculus AB
Monday, May 6, 2013

Two days!

$f(x)=e^{2x}$

Come here for ETEH today & tomorrow!

Evaluate $\lim_{h\to 0} \frac{e^{2(-1+h)}-e^{2(-1)}}{h}$

$f'(-1)$ of $f(x)=e^{2x}$

$f'(x)=2e^{2x}$

$f'(-1)=2e^{-2}$

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

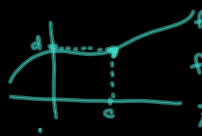
Def'n of Continuity:

In order for a function to be cont @ a point (c),

① $f(c)$ exists

② $\lim_{x\to c} f(x)$ exists

③ $\lim_{x\to c} f(x)=f(c)$.

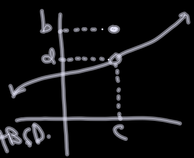


$f(c)=d$ (exists)

$\lim_{x\to c} f(x)=d$

$\lim_{x\to c} f(x)=f(c)$

$\therefore f(x)$ is cont. @ c



ABSD.

$f(c)=b$

$\lim_{x\to c} f(x)=d$

$\lim_{x\to c} f(x)\neq f(c)$ $\therefore$ $f(x)$ is NOT cont @ $x=c$.

Ex. $f(x)=\begin{cases}\sqrt{1-x}, & x\le 1\\ kx+2, & x>1\end{cases}$

What value of k will make $f(x)$ cont @ $x=1$

$\sqrt{1-1}=k(1)+2$

$0=k+2$

$k=-2$

$f(x)=\begin{cases}\sqrt{1-x}, & x\le 1\\ -2x+2, & x>1\end{cases}$

$\lim_{x\to 2}\frac{x-2}{x^3-8}=\frac{0}{0}\to$ try something else

$$\begin{array}{r|l} & x^2+2x+4 \\ x-2 & x^3+0x^2+0x-8\end{array}$$

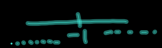
$$\begin{array}{c|cccc} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & \boxed{0}\end{array}$$

$\lim_{x\to 2}\frac{\cancel{x-2}}{(\cancel{x-2})(x^2+2x+4)}=\lim_{x\to 2}\frac{1}{x^2+2x+4}=\frac{1}{1}$

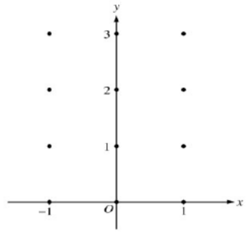
L'Hopital's Rule

$\lim_{x\to 2}\frac{x-2}{x^3-8}=\lim_{x\to 2}\frac{1}{3x^2}=\frac{1}{1}$

$\lim_{x\to -1}\frac{1}{x+1}=\frac{1}{0}$



Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.



(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

$\frac{dy}{dx} > 0$? $\quad x^2 \neq 0$

$x \neq 0$

$y - 1 > 0$

$y > 1$

$$\frac{dy}{dx} = \frac{x^2(y-1)}{1}$$

$$\ln u = \frac{u'}{u}$$

$$\int \frac{dy}{y-1} = \int x^2 dx$$

$$\int \frac{1}{x+2}$$

$$\ln \boxed{|y-1|} = \frac{x^3}{3} + C$$

>0

$$\int \frac{2}{3x-1}$$

$$f(0) = 3$$

$$\ln|3-1| = \frac{0^3}{3} + C$$

$$\int \frac{2}{3}\left(\frac{1}{x-\frac{1}{3}}\right)dx$$

$$\ln 2 = C$$

$$\ln|y-1| = \frac{x^3}{3} + \ln 2$$

$$e^{(x^3/3 + \ln 2)} = |y-1|$$

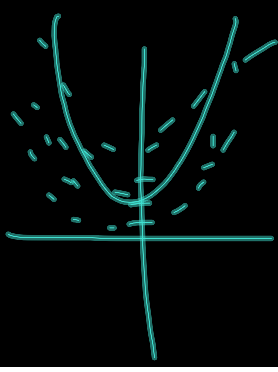
$$1 + e^{(x^3/3 + \ln 2)} = y$$

$$1 + 2e^{x^3/3} = y$$

$$\frac{dy}{dx} = 2x$$

$$\int dy = \int 2x\,dx$$

$$y = x^2 + C$$



$$\frac{dy}{dx} = 1 + \sin y$$

$$\rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} \cos y$$

$$\rightarrow \frac{d^2y}{dx^2} = (1+\sin y)\cos y$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$\int y\,dy = \int -x\,dx$$

$$\frac{y^2}{2} = \frac{-x^2}{2} + C$$

point $(1,-2)$

$$\frac{(-2)^2}{2} = \frac{-(1)^2}{2} + C$$

$$\frac{4}{2} = -\frac{1}{2} + C$$

$$C = \frac{5}{2}$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + \frac{5}{2}$$

solve for y.

$$y^2 = -x^2 + 5$$

$$y = \pm\sqrt{5-x^2}$$

Decide by using $(1,-2) \rightarrow$ given

$$y = -\sqrt{5-x^2}$$

$$d(x) = \int_3^{2x^2} (t^3 - 3t + 4)\,dt$$

$$d'(x) = \left[(2x^2)^3 - 3(2x^2) + 4\right] \cdot 4x$$

$$d'(x) = 4x(8x^6 - 6x^2 + 4)$$

$$m(x) = \int_x^5 (t^3 - 4t + 6)\,dt$$

$$m'(x) = -(x^3 - 4x + 6)$$