AP Calculus AB
Tuesday, April 29, 2014

ETEH This week...starting tomorrow

We will finish going over the problem from Friday & we will do a separation of variables problem

Homework: Finish 2013 free response

$G(t) \to$ is A RATE gravel is arriving

$G(5) = 98.1407$ is the rate @ which the gravel arrives. The processing rate is given to be 100 tons/hr.

$98.1407 < 100$ ∴ the amount of unprocessed grav is decreasing @ $t = 5$.

d) 500 tons is present @ $t = 0$

Rate of change of gravel @ plant is

$(G(t) - 100)\ \frac{\text{tons}}{\text{hr}}$

$A(t)$ = amount of unprocessed gravel

$A(t) = 500 + \int_0^t (G(t) - 100)$

First find when $A(t)$ is a maximu

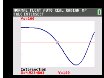
$A'(t) = 0$

$A'(t) = G(t) - 100$

use calc →

$G(t) - 100 = 0$

OR $G(t) = 100$



This occurs @ $t = 4.92$

t	$A(t) = 500 + \int_0^t (G($
0	500
4.9234803	635.3761
8	525.551

The maximum amount of unprocesse will occur @ an endpoint first val OR critical value. The maximum amount of g 635.3761 tons.

$\frac{dy}{dx} = \frac{e^y(3x^2 - 6x)}{1}$

b)

$dy = e^y(3x^2 - 6x)\,dx$

$\frac{dy}{e^y} = (3x^2 - 6x)\,dx$

$\int e^{-y}\,dy = \int (3x^2 - 6x)$

$u = -y$

$du = -dy$

$-du = dy$

$-\int e^u\,du = \int (3x^2 - 6x)\,dx$

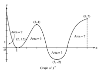
$-e^u = x^3 - 3x^2 + C$

$-e^{-y} = x^3 - 3x^2 + C$

$e^{-y} = -x^3 + 3x^2 - C$

Question 4

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.



Graph of f'

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

(a) At $x=6$, f has a local minimum because $f'(x)$ goes from negative to positive @ $x=6$.

(b) $f(x) = \int f'(x)dx$

	Area	Accumulated Area
$0 \to 1$	2	2
$1 \to 4$	6	8
$4 \to 6$	-3	5
$6 \to 8$	7	12

Absolute minimum will occur @ an endpoint or a local minimum. We need to consider $x=0$, $x=6$, & $x=8$.

$f(x) = \int_0^x f'(t)dt$

$\int_0^8 f'(t)dt = f(8) - \boxed{f(0)}$ ← This allows us to find $f(0)$.

$f(8) = 4$ (Given)

$f(0) = f(8) - \int_0^8 f'(t)dt$

$f(0) = 4 - 12$

$f(0) = -8$

Now find $f(6)$.

$\int_6^8 f'(t)dt = f(8) - f(6)$

$f(6) = f(8) - \int_6^8 f'(t)dt$

$f(6) = 4 - 7$

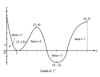
$f(6) = -3$

Summarizing:

$f(0) = -8 \longrightarrow -8$ is the absolute minimum on $0 \le x \le 8$.

$f(6) = -3$

$f(8) = 4$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Graph of f'

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(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

(c) f will be concave down when $f'' < 0$ or f' is decreasing.

$0 < x < 1$ & $3 < x < 5$

f will be increasing when $f' > 0$.

f is both concave down & increasing when $0 < x < 1$ & $3 < x < 4$.

(d) $g(x) = [f(x)]^3$

$g'(x) = 3(f(x))^2 \cdot f'(x)$

$g'(3) = 3(f(3))^2 \cdot f'(3)$

$= 3\left(-\frac{5}{2}\right)^2 \cdot 4$ ← from graph

$= 3 \cdot \frac{25}{4} \cdot 4$

$= 75$

Question 3

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

(a) $C'(3.5) = \frac{C(4) - C(3)}{4 - 3} = \frac{12.8 - 11.2}{1}$ $\frac{oz}{min}$

(b) MVT

$C(t)$ is continuous & differentiable

$\frac{C(4) - C(2)}{4 - 2} = \frac{12.8 - 8.8}{2} = \frac{4}{2} = 2$

By the MVT, there exists a value between 2 & 4 such that $C'(t) = 2$.