AP Calculus AB

Friday, April 26, 2013

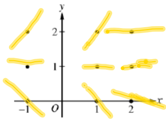
You should study for AT LEAST one hour every day this weekend.

Next weekend...study here?

**Question 5**

Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
**(Note: Use the axes provided in the exam booklet.)**



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x\to\infty} f(x)$.

$$\frac{dy}{dx} = \frac{y-1}{x^2}$$

$$dy\, x^2 = dx\, y-1$$

$$\int dy\,(y-1)^{-1} = \int dx\,(x^2)^{-1}$$

$$\int \boxed{\frac{1}{y-1}}\,dy \rightarrow \ln$$

$$\frac{u'}{u}$$

$$\ln|y-1| = \frac{-1}{x} + c$$

$$\ln_e|y-1| = -\frac{1}{x} + C$$

exp. form

$f(2)=0$ particular solution

$$e^{\left(-\frac{1}{x}+c\right)} = |y-1|$$

prop of exp

$$e^{-\frac{1}{x}} \cdot e^{c} = |y-1|$$

$f(2)=0$

$$e^{-\frac{1}{2}} \cdot e^{c} = |0-1|$$

$$e^{-\frac{1}{2}} \cdot e^{c} = 1$$

$$e^{c} = e^{\frac{1}{2}}$$

$$C = \frac{1}{2}$$

$$e^{-\frac{1}{x}} \cdot e^{\frac{1}{2}} = |y-1|$$

$$e^{-\frac{1}{x}+\frac{1}{2}} = |y-1|$$

$$e^{-\frac{1}{x}+\frac{1}{2}} = y-1 \quad \text{OR} \quad e^{-\frac{1}{x}+\frac{1}{2}} = -y+1 \quad ?$$

use $f(2)=0$ to determine which one

$$y = 1 + e^{-\frac{1}{x}+\frac{1}{2}}$$

Plug in $f(2)=0$?

No.

$$y = 1 - e^{-\frac{1}{x}+\frac{1}{2}}$$

$$1 - e^{-\frac{1}{2}+\frac{1}{2}}$$

$$1 - e^{0}$$

$$1 - 1$$

$$0$$

(c) $\lim_{x\to\infty}\left(1 - e^{-\frac{1}{x}+\frac{1}{2}}\right)$

$$= 1 - e^{\frac{1}{2}}$$

Let $f$ be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of $f$ is given by $f'(x) = \frac{1 - \ln x}{x^2}$.

(a) Write an equation for the line tangent to the graph of $f$ at $x = e^2$.

(b) Find the $x$-coordinate of the critical point of $f$. Determine whether this point is a relative minimum, a relative maximum, or neither for the function $f$. Justify your answer.

(c) The graph of the function $f$ has exactly one point of inflection. Find the $x$-coordinate of this point.

(d) Find $\lim_{x \to 0^+} f(x)$.

(a) point $\rightarrow f(e^2) = \frac{\ln e^2}{e^2} = \frac{2}{e^2}$

slope $\rightarrow f'(e^2) = \frac{1 - \ln e^2}{e^4} = \frac{-1}{e^4}$

$y - y_1 = m(x - x_1)$

$y - \frac{2}{e^2} = \frac{-1}{e^4}(x - e^2)$

(b) Critical pt $\rightarrow f'(x) = 0$ also $f'(x)$ und.

$f'(x) = \frac{1 - \ln x}{x^2}$

$1 - \ln x = 0$

$\ln x = 1$

$x = e$

$f'(x)$ is undefined @ $x = 0$ but $x = 0$ is not in domain.

$+ \quad -$

$0 \quad e^0 \quad e^1 \quad e^2$

Since $f'(x)$ goes from positive to negative @ $x = e$, there is a relative maximum @ $x = e$.

$$\frac{1}{5}\int \frac{5\cdot 1}{5x-3}\,dx$$

$u = 5x-3$

$du = 5dx$

$$\frac{1}{5}\int \frac{du}{u}$$

$\frac{1}{5}\ln|u| + C$

$\frac{1}{5}\ln|5x-3| + C$

$$\frac{1}{2}\int \frac{2x}{x^2-1}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|x^2-1| + C$$

$u = x^2-1$

$du = 2x\,dx$

$$\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$$\int \frac{du}{u} = \ln|u| + C$$

$$= \ln|\sin x| + C$$