AP Calculus AB

Wednesday, April 10, 2013

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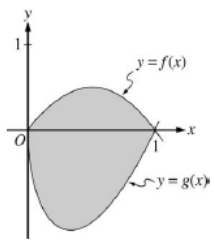
Volume of Solids

MMM #34

AP Free Response Question



2. Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

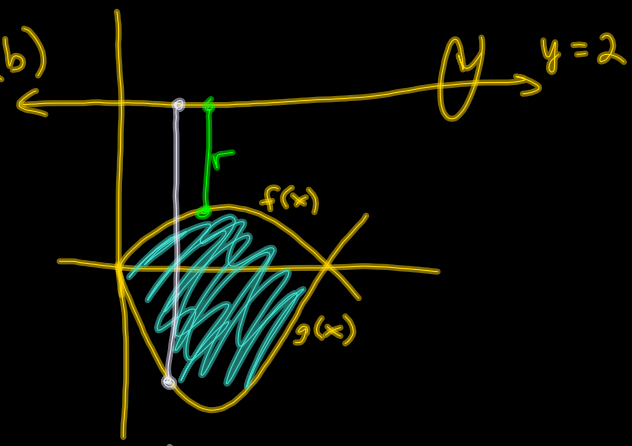
(c) Let h be the function given by $h(x) = kx(1-x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

(a) $\int_0^1 [f(x) - g(x)]\,dx \approx 1.1333$

Lose ↓

$\int_0^1 [2x(1-x) - 3x - 1\sqrt{x})\,dx \approx 1.1333$

(b)



$R = 2 - g(x)$

$r = 2 - f(x)$

$\pi \int_0^1 \left[(2-g(x))^2 - (2-f(x))^2\right] dx \approx 16.179$

Draw a diagram for each scenario. Set up the integrals. YOU DO NOT HAVE TO EVALUATE THE INTEGRALS.

1. Find the volume if the region enclosing $y = 4 - x, x = 0, y = 0$ is rotated about the

a) x-axis

R = _____ r = _____

V = _____

b) the line $y = 4$

R = _____ r = _____

V = _____

c) the line $y = 5$

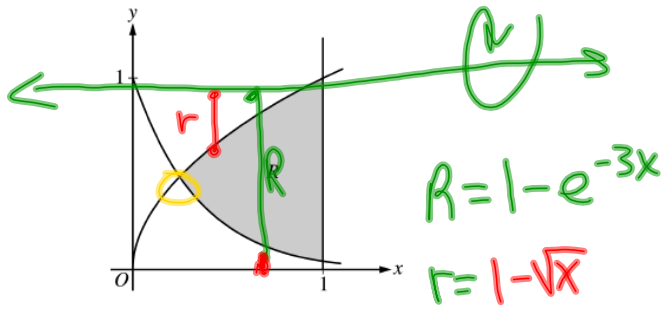
R = _____ r = _____

V = _____

d) the y-axis

e) the line $x = 4$

f) the line $x = 6$



$R = 1 - e^{-3x}$

$r = 1 - \sqrt{x}$

1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

$\sqrt{x} = e^{-3x}$

(b) $\pi\int_{0.2387\ldots}^{1}\left[(1-e^{-3x})^2-(1-\sqrt{x})^2\right]dx \approx 1.424$

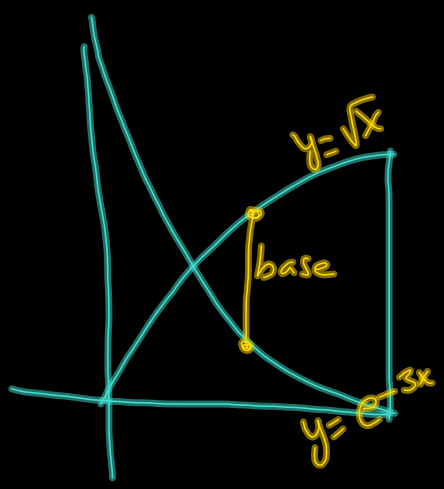
point of intersection

OR

x-value of pt of int.

$\int_A^1 (\sqrt{x} - e^{-3x})\,dx = 0.443$

$A \approx 0.2387$



base $= \sqrt{x} - e^{-3x}$

ht $= 5(\sqrt{x} - e^{-3x})$

$\int_{0.239}^{1}\left[5(\sqrt{x}-e^{-3x})(\sqrt{x}-e^{-3x})\right]dx$

Ex 1

(a) 36π (b) 72π (c) 108π

(d) 36π (e) 18π (f) 36π

Ex 2

(a) $\frac{8\pi}{3}$ (b) $\frac{16\pi}{3}$ (c) $\frac{28\pi}{3}$

(d) $\frac{28\pi}{3}$ (e) $\frac{8\pi}{3}$ (f) $\frac{32\pi}{3}$

(a) $R = x-1$

(b) $R=2$
$r=2-(x-1)$

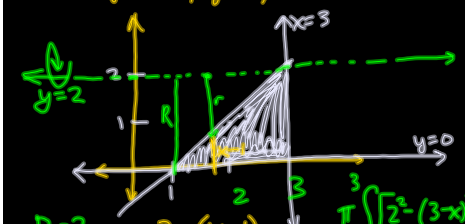
(c) $R=3$
$r=3-(x-1)$

(d) $R=3$
$r=y+1$

(e) $R=2-y$

(f) $R=4-y$
$r=2$

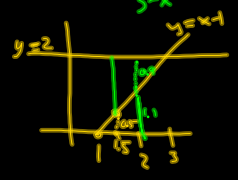
(2b) $y=x-1,\ y=0,\ x=3$



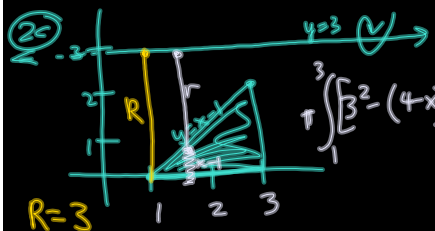
$R=2 \quad r=2-(x-1)$

$3-x$

$$\pi\int_1^3\left[2^2-(3-x)^2\right]dx$$



(2c)

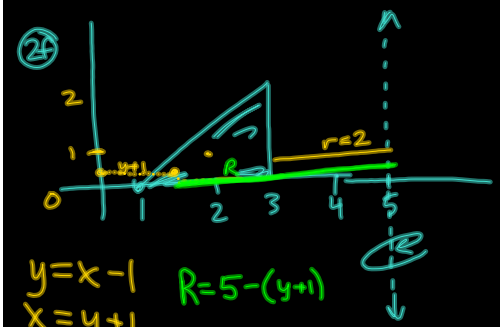


$$\pi\int_1^3\left[3^2-(4-x)^2\right]dx$$

$R=3$

$r=3-(x-1)$

(2f)



$y=x-1 \qquad R=5-(y+1)$

$x=y+1$

$$\pi\int_0^2\left[(4-y)^2-2^2\right]dy$$

$$\pi\int_0^2\left(16-8y+y^2-4\right)dy$$

$$\pi\int_0^2\left(12-8y+y^2\right)dy$$

$$=\pi\left[12y-\frac{8y^2}{2}+\frac{y^3}{3}\right]\Big|_0^2$$

$$=\pi\left[12y-4y^2+\frac{y^3}{3}\right]\Big|_0^2$$

$$=\pi\left[12\cdot 2-4\cdot 2^2+\frac{2^3}{3}-0\right]$$

$$=\pi\left[24-16+8\right]$$