9. Find the volume of the solid whose base is bounded by the circle whose center is the origin and whose radius is 10 with the indicated cross sections perpendicular to the x-axis

a) squares

$$A=(2y)^2 \quad V=8\int_0^{10}\left(100-x^2\right)dx=\frac{16000}{3}$$

b) equilateral triangles

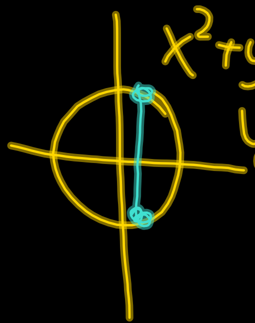
$$A=\frac{1}{2}(2y)\left(y\sqrt{3}\right) \quad V=2\sqrt{3}\int_0^{10}\left(100-x^2\right)dx=2309.401$$

c) semi-circles

$$A=\frac{1}{2}\pi y^2 \quad V=\pi\int_0^{10}\left(100-x^2\right)dx=2094.395$$

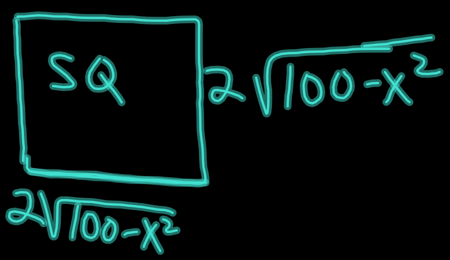
d) isosceles right triangles

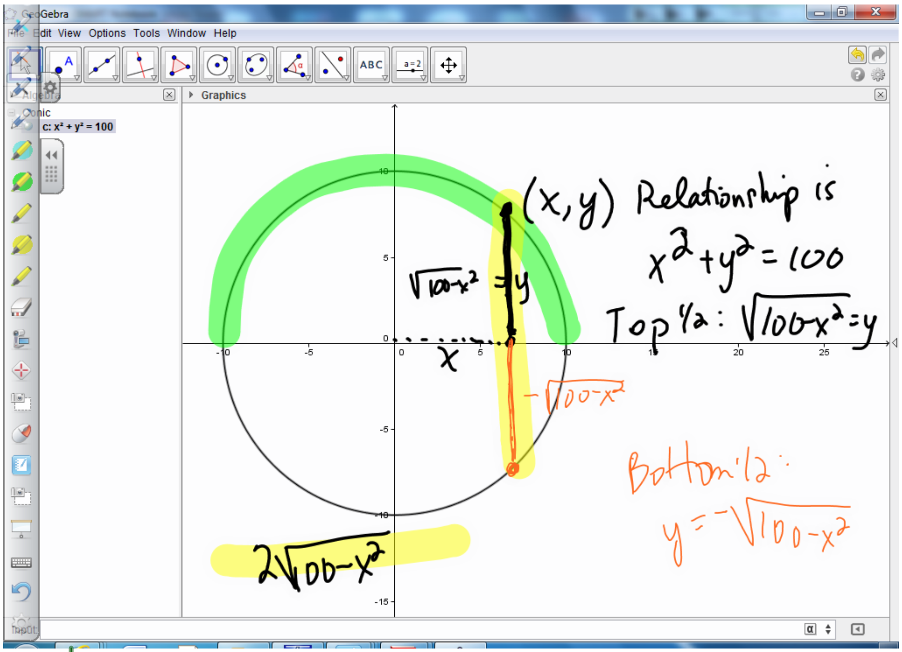
$$A=\frac{1}{2}(2y)(y) \quad V=2\int_0^{10}\left(100-x^2\right)dx=\frac{4000}{3}$$



$x^2+y^2=100$

$y=\sqrt{100-x^2}$





$$\int_a^b \left[\text{area of cross section}\right] dx$$

dx → width of cross section

square:

$$\int_{-10}^{10} \left(2\sqrt{100-x^2}\right)^2 dx$$

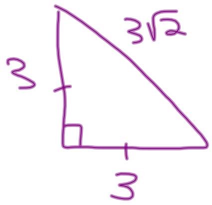
Symmetry:

$$2\int_0^{10} 4(100-x^2)\,dx$$

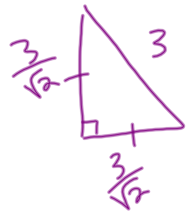
$$8\int_0^{10} (100-x^2)\,dx$$

$$8\left[100x-\frac{x^3}{3}\right]\Bigg|_0^{10}$$

$$= 8\left(100\cdot 10-\frac{10^3}{3}-0\right)$$

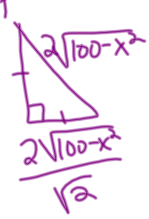


$$\text{area} = \frac{1}{2} \cdot 3^2 = \frac{9}{2}$$



$$\text{area} = \frac{1}{2} \cdot \frac{9}{2} = \frac{9}{4}$$

Hypotenuse as base:



$$\text{area} = \frac{1}{2}\left(\frac{2\sqrt{100-x^2}}{\sqrt{2}}\right)^2$$

$$\text{area} = \frac{1}{2}\left(\frac{4(100-x^2)}{2}\right)$$

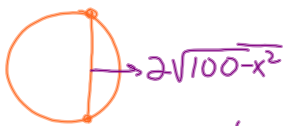
$$\text{area} = 100-x^2$$

$$\text{Volume: } 2\int_0^{10}(100-x^2)\,dx$$

$$= 2\left[100x - \frac{x^3}{3}\right]\Bigg|_0^{10}$$

$$= 2\left[100 \cdot 10 - \frac{10^3}{3}\right]$$

semi-circles



diameter of semicircle $= 2\sqrt{100-x^2}$

radius $= \sqrt{100-x^2}$

area of a circle $= \pi r^2$

area of SC : $\frac{\pi r^2}{2}$

$$\cancel{2}\int_0^{10} \frac{\pi\left(\sqrt{100-x^2}\right)^2}{\cancel{2}}\,dx$$

$$\pi \int_0^{10}(100-x^2)\,dx$$