AP Calculus AB
Tuesday, December 3, 2013

Please work on #3 from the Free Response Packet.

Also...Take a picture of your HW and then turn it into the tray.

3) Consider the curve $xy^2 - x^3y = 6$

a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$

$$x \cdot 2y \cdot y' + y^2 \cdot 1 - (x^3 y' + y \cdot 3x^2 \cdot 1) = 0$$

$$2xy \cdot y' + y^2 - x^3 y' - 3x^2 y = 0$$

$$y'(2xy - x^3) = 3x^2y - y^2$$

$$y' = \frac{3x^2y - y^2}{2xy - x^3}$$

b) Find all points on the curve whose x-coordinate is 1, and write the equation for the tangent line at these points.

$$xy^2 - x^3y = 6$$

Plug in $x = 1$ & find y

$$1 \cdot y^2 - 1^3 y = 6$$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0$$

$$y = 3 \quad y = -2$$

$$(1,3) \quad (1,-2)$$

Now, find slope @ these points:

@ $(1,3)$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

$$\left.\frac{dy}{dx}\right|_{(1,3)} = \frac{3 \cdot 1^2 \cdot 3 - 3^2}{2 \cdot 1 \cdot 3 - 1^3} = 0$$

@ $(1,-2)$

$$\left.\frac{dy}{dx}\right|_{(1,-2)} = \frac{3 \cdot 1^2(-2) - (-2)^2}{2 \cdot 1(-2) - (1)^3}$$

$$= \frac{-6-4}{-4-1} = 2$$

c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

$$2xy - x^3 = 0$$

$$x(2y - x^2) = 0$$

$$x = 0 \quad 2y - x^2 = 0$$

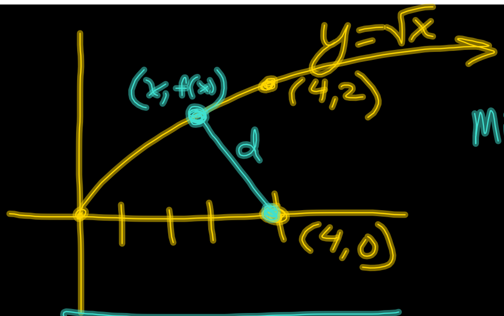
$$x^2 = 2y$$

$$\rightarrow x = \pm\sqrt{2y}$$

$$xy^2 - x^3y = 6$$

Find the point on the graph of the function that is closest to the given point.

Function	*Point*
$f(x) = \sqrt{x}$	$(4, 0)$



Goal:
Minimize d

$d = \sqrt{(x-4)^2 + (f(x)-0)^2}$

$d = \sqrt{(x-4)^2 + (\sqrt{x}-0)^2}$

$d = \sqrt{x^2 - 8x + 16 + x}$

$d = (x^2 - 7x + 16)^{1/2}$

Find minimum: $x^2 - 7x + 16 = g(x)$

$g'(x) = 2x - 7$

$g'(x) = 0$ when $x = \frac{7}{2}$

Absolute minimum

x	$d = \sqrt{x^2 - 7x + 16}$
0	4
$\frac{7}{2}$	1.936
4	2
5	$\sqrt{25 - 35 + 16} = \sqrt{6} > 2$

Tonight - Try last night's hw (again)

- FR packet
- deriv rules
- unit circle + trig