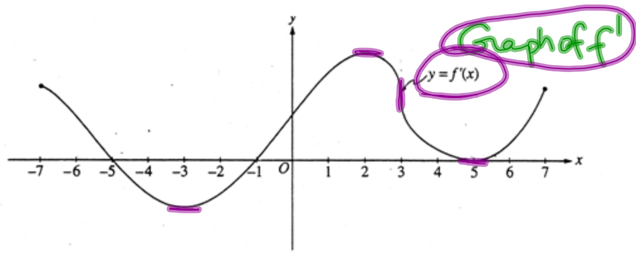
1)



The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

a) f attains a relative minimum when f' goes from negative to positive. This occurs @ $x = -1$.

b) f attains a relative maximum when f' goes from positive to negative. This occurs @ $x = -5$.

c) $f''(x) < 0$ when f' is decreasing. This occurs on the following intervals: $(-7,-3) \cup (2,3) \cup (3,5)$.

(5 points) 1. Suppose a function f is differentiable for all x and $f(0)=0$. If $g(x)$ is defined as $g(x) = f(x)\cos(x)$, which of the following statements must be true? (Circle your answer.)

I. There exists a number c in $\left(0, \frac{\pi}{2}\right)$ such that $g'(c) = 0$.

~~II. There exists a number c in $\left(\frac{\pi}{2}, \pi\right)$ such that $g'(c) = 0$.~~

III. There exists a number c in $\left(-\frac{\pi}{2}, 0\right)$ such that $g'(c) = 0$.

① Given $f(0)=0 \rightarrow g(0)=f(0)\cos(0)$

$g(0)=0$

$g(\frac{\pi}{2}) = f(\frac{\pi}{2})\cos\frac{\pi}{2}$

$g(\frac{\pi}{2})=0$

slope btn $(0,0)$ & $(\frac{\pi}{2},0)$ is 0.

$g(-\frac{\pi}{2}) = f(-\frac{\pi}{2}) \cdot \cos(-\frac{\pi}{2})$

$g(-\frac{\pi}{2})=0$

Slope btn $(-\frac{\pi}{2},0)$ & $(0,0)$ is 0.

④ $f(x)=\sqrt{1-x^2}$

1^{st}: Check for C & D.

$f(x)$ is continuous on $[0,1]$ and differentiable on $(0,1)$.

$\therefore$ we can use MVT.

$f(0)=1$

$f(1)=0$

$$m_{sec}=\frac{f(1)-f(0)}{1-0}=\frac{0-1}{1-0}=-1$$

$$f(x)=(1-x^2)^{1/2}$$

$$f'(x)=\tfrac{1}{2}(1-x^2)^{-1/2}(-2x)$$

$$f'(x)=\frac{-x}{\sqrt{1-x^2}}$$

$$\frac{-x}{\sqrt{1-x^2}}=-1$$

$$\frac{x}{\sqrt{1-x^2}}=\frac{1}{1}$$

$$x=\sqrt{1-x^2}$$

$$x^2=1-x^2$$

$$2x^2=1$$

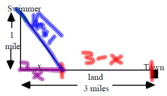
$$x^2=\tfrac{1}{2}$$

$$x=\pm\sqrt{\tfrac{1}{2}}$$

only use $+\sqrt{\frac{1}{2}}$ bc it is on $[0,1]$.

Example 5) I am 1 mile in the ocean and wish to get to a town 3 miles down the coast which is very rocky. I need to swim to the shore and then walk along the shore. What point should I swim to along the shoreline so that the time it takes to get to town is a minimum? I swim at 2 mph and walk at 4 mph.

Primary



	distance	rate	time
swim	$\sqrt{x^2+1}$	2	$\frac{1}{2}\sqrt{x^2+1}$
walking	$3-x$	4	$\frac{1}{4}(3-x)$

$d=rt$

$\frac{d}{r}=t$

$$\boxed{f(x)=\tfrac{1}{2}\sqrt{x^2+1}+\tfrac{1}{4}(3-x)}$$ Minimize

Rewrite then find $f'(x)$.

$$f(x)=\tfrac{1}{2}(x^2+1)^{1/2}+\tfrac{3}{4}-\tfrac{1}{4}x$$

$$f'(x)=\tfrac{1}{4}(x^2+1)^{-1/2}(2x)-\tfrac{1}{4}$$

$$\tfrac{1}{4}(x^2+1)^{-1/2}(2x)-\tfrac{1}{4}=0$$

$$\frac{2x}{\sqrt{x^2+1}}=1$$

$$2x=\sqrt{x^2+1}$$

$$4x^2=x^2+1$$

$$3x^2=1$$

$$x^2=1/3$$

$$x=\pm\sqrt{\tfrac{1}{3}}$$

$f'(x)=\frac{1}{4}(x^2+1)^{-1/2}\cdot(2x)-\frac{1}{4}$

⊖ $\frac{1}{2}$ ⊕

0 $\sqrt{\frac{1}{3}}$

$\frac{1}{4}\cdot\left(\frac{5}{4}\right)^{-1/2}\cdot 2\cdot\frac{1}{2}-\frac{1}{4}$

$\frac{\frac{1}{4}}{\sqrt{5/4}}$

$\frac{\frac{1}{4}}{\frac{\sqrt{5}}{2}}$

$\frac{1}{4}\cdot\frac{2}{\sqrt{5}}=\frac{1}{2\sqrt{5}}$

$\frac{1}{>4}-\frac{1}{4}<0$

$\cancel{\frac{1}{4}}(2)^{-1/2}\cdot\cancel{4}-$

$\frac{1}{\sqrt{2}}-\frac{1}{4}>$

From this analysis, $\sqrt{}$ is a relative minimum. But we are looking for absolute minimum. We need to check end[points] also.

x	$f(x)=\frac{1}{2}\sqrt{x^2+1}-\frac{1}{4}(3-x)$
0	
$\sqrt{1/3}$	
3	