

Be Prepared  
 p 318 #16



$$A = \pi r^2$$

$$V = \pi r^2 h$$

$$\frac{dA}{dt} = 3 \text{ m}^2/\text{min}$$

$$A = 100\pi \text{ m}^2$$

$$r = 10 \text{ m}$$

(a)  $\frac{dA}{dt} = ?$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi \cdot 10 \cdot 3 \text{ m}^2/\text{min}$$

$$\frac{dA}{dt} = 60\pi \frac{\text{m}^2}{\text{min}}$$

(b)  $V = \pi r^2 h$

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + h \cdot 2\pi r \cdot \frac{dr}{dt}$$

We know  $V = 100000 \text{ m}^3 \rightarrow \frac{dV}{dt} = 0 \frac{\text{m}^3}{\text{min}}$

$$r = 10 \text{ m}$$

$$\frac{dr}{dt} = 3 \text{ m}/\text{min}$$

$\frac{dh}{dt} = ?$

$$0 = \pi \cdot 10^2 \cdot \frac{dh}{dt} + h \cdot 2\pi \cdot 10 \cdot 3$$

$$V = \pi r^2 h$$

$$100000 = \pi \cdot 10^2 \cdot h$$

$$h = \frac{10000}{\pi} \text{ m}$$

$$0 = \pi \cdot 100 \cdot \frac{dh}{dt} + \frac{10000}{\pi} \cdot 2\pi \cdot 30$$

$$-100\pi \frac{dh}{dt} = 60000$$

$$\frac{dh}{dt} = \frac{60000}{-100\pi} \text{ m}/\text{min} \text{ OR } -\frac{600}{\pi} \frac{\text{m}}{\text{min}}$$

(c)  $\frac{dA}{dh} = ?$

$$V = \pi r^2 h$$

$$V = Ah$$

$$\frac{dV}{dt} = A \frac{dh}{dt} + h \cdot \frac{dA}{dt}$$

$$0 = A \frac{dh}{dt} + h \frac{dA}{dt}$$

manipulate eqn to find  $\frac{dA}{dh}$

$$-A \frac{dh}{dt} = h \frac{dA}{dt}$$

$$-\frac{A dh}{dt} = \frac{h \cdot dA}{dt}$$

$$-\frac{A dh}{dh} = \frac{h dA}{dh}$$

$$-A = h \frac{dA}{dh}$$

$$-\frac{A}{h} = \frac{dA}{dh}$$

$$\frac{dA}{dh} = \frac{-100\pi}{\frac{10000}{\pi}} \frac{\text{m}^2}{\text{m}}$$

$$\frac{dA}{dh} = \frac{dh}{dt} \cdot \frac{dA}{dh}$$

$$\frac{dA}{dh} = 60\pi \div \frac{600}{\pi}$$

$$= 60\pi \cdot \frac{\pi}{600}$$

Math: 8

nDeriv( $y$ ,  $x$ ,  $x$ -value)

nDeriv (function, <sup>x</sup>variable, value)

$y_1 =$

Ex.  $y = x^2$

Use calc to graph  $y'$ :

$$y_1 = x^2$$

$$y_2 = \text{nDeriv}(y, x, x)$$

OR find value of deriv @  $x = -1$ :

homework:

$$\text{nDeriv}(x^2, x, -1)$$

$$y' = 2x$$

$$y' \text{ at } x = -1 \rightarrow -2$$

$$f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$$

$$y_1 = (\cos(x))^2 \div x - 0.2$$

$$x: 0 \text{ to } 10$$

$$y: -1 \text{ to } 1$$

$$\textcircled{21} \lim_{x \rightarrow \infty} \frac{x^2 - 4}{2x + 4x^2} = -\frac{1}{4}$$

$$\textcircled{22} \lim_{h \rightarrow 0} \frac{\cos\left(\frac{\pi}{2} + h\right) - \cos\frac{\pi}{2}}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = \cos x \quad \left\{ \begin{array}{l} \text{Thus question is} \\ \text{asking for } f'\left(\frac{\pi}{2}\right)? \end{array} \right.$$

x-value is  $\frac{\pi}{2}$

$$f'(x) = -\sin x$$

$$f'\left(\frac{\pi}{2}\right) = -1$$

$$\textcircled{40} \text{ find } \frac{dV}{dt} \text{ ; plug in } 40 \text{ sec.}$$

$$\frac{dV}{dt} = -40 + 0.4t$$

$$\left. \frac{dV}{dt} \right|_{t=40} = -40 + 0.4(40)$$

$$= -24 \text{ m}^3/\text{sec}$$

† function is continuous @  $x=c$  iff:

- ①  $f(c)$  exists
  - ②  $\lim_{x \rightarrow c} f(x)$  exists
  - ③  $\lim_{x \rightarrow c} f(x) = f(c)$
- lim from left =  
lim from right

L'Hopital's Rule

$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \infty} \frac{f'(x)}{g'(x)}$$

Ex:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{0}{0}$  indeterminate form

use L'Hopital's Rule:

$$\lim_{x \rightarrow 0} \frac{\cos x}{1} = \frac{1}{1} = 1$$