

1. $\lim_{h \rightarrow 0} \frac{\ln(4+h) - \ln(4)}{h}$ is

2. Find $f'(4)$ if $f(x) = \ln x$. $f'(x) = \frac{1}{x}$
 $f'(4) = \frac{1}{4}$

These are exactly the same problems.

Defn of Derivative

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

4) Let p and q be real numbers and let f be the function defined by

$$f(x) = \begin{cases} 1 + 2p(x-1) + (x-1)^2 & x \leq 1 \\ qx + p & x > 1 \end{cases} \rightarrow \frac{d}{dx} \rightarrow 0 + 2p + 2(x-1) \cdot 1$$

- Find the values of q , in terms of p , for which f is continuous at $x = 1$
- Find the values of p and q for which f is differentiable at $x = 1$
- If p and q have the values determined in part b, is f'' a continuous function? Justify your answer.

a) $1 + 2p(1-1) + (1-1)^2 = q \cdot 1 + p$

$$1 = q + p$$

$$\star 1 - p = q$$

b) $2p + 2(x-1) = q$

$$2p + 2(1-1) = q$$

$$\star 2p = q$$

$$1 - p = q$$

$$2p = 1 - p$$

$$3p = 1$$

$$p = \frac{1}{3} \rightarrow q = \frac{2}{3}$$

c) $f(x) = \begin{cases} 1 + 2 \cdot \frac{1}{3}(x-1) + (x-1)^2, & x \leq 1 \\ \frac{2}{3}x + \frac{1}{3}, & x > 1 \end{cases}$

$$f'(x) = \begin{cases} \frac{2}{3} + 2(x-1), & x \leq 1 \\ \frac{2}{3}, & x > 1 \end{cases}$$

$$f''(x) = \begin{cases} 2, & x \leq 1 \\ 0, & x > 1 \end{cases}$$

The second derivative is not continuous at $x = 1$.

$$\lim_{x \rightarrow 1^-} f''(x) = 2$$

$$\lim_{x \rightarrow 1^+} f''(x) = 0$$

5)

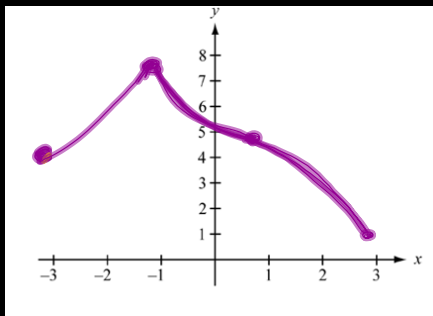
A function f is continuous on the closed interval $[-3, 3]$ such that $f(-3) = 4$ and $f(3) = 1$. The functions f' and f'' have the properties given in the table below.

x	$-3 < x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$
$f'(x)$	Positive	Fails to exist	Negative	0	Negative
$f''(x)$	Positive	Fails to exist	Positive	0	Negative

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- What are the x -coordinates of all absolute maximum and absolute minimum points of f on the interval $[-3, 3]$? Justify your answer.
- What are the x -coordinates of all points of inflection of f on the interval $[-3, 3]$? Justify your answer.
- On the axes provided, sketch a graph that satisfies the given properties of f .

(b) A point of inflection occurs when $f''(x)$ goes from positive to negative or negative to positive. According to the chart, $f''(x)$ goes from positive to negative @ $x=1$. Therefore, $f(x)$ has a point of inflection at the x -value of 1.



At $x = -1$, $f(x)$ has an absolute maximum because $f'(x)$ goes from positive to negative @ $x = -1$ and $f(-3) = 4$. Since $f'(x)$ is positive from $x = -3$ to $x = -1$, $f(x)$ is increasing on $(-3, -1) \therefore f(-1) > f(-3)$ making $x = -1$ the x -value for the absolute maximum.

Since $f'(x)$ is negative on $(-1, 1)$ and $(1, 3)$, $f(x)$ is decreasing on those intervals. Therefore, $f(x)$ has an absolute minimum at $x = 3$.