AP Calculus AB
Wednesday, December 11, 2013

13) The slope of the tangent line to the graph of

$$4x^2 + cx + 2e^y = 2$$

at $x = 0$ is 4. Give the value of c.

1st find derivative

$\frac{d}{dx}[e^y] = y'e^y$

$4x^2 + cx + 2e^y = 2$

$8x + c + 2y'e^y = 0$

$x = 0$

$4 \cdot 0^2 + c \cdot 0 + 2e^y = 2$

$2e^y = 2$

$e^y = 1$ $y = 0$

Solve for y':

$2y'e^y = -c - 8x$

$y' = \frac{-c-8x}{2e^y}$

@ point $(0,0)$, $y' = 4$

$4 = \frac{-c - 8 \cdot 0}{2e^0}$

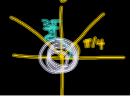
$4 = -\frac{c}{2}$

$c = -8$

$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\sin x = \frac{\sqrt{2}}{2}$

What angle has a sine of $\sqrt{2}/2$?



$y = \sin^{-1} x$ if and only if $\sin y = x$ and $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$y = \cos^{-1} x$ if and only if $\cos y = x$ and $y \in [0, \pi]$

$y = \tan^{-1} x$ if and only if $\tan y = x$ and $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Principal Values for Inverse Trig Functions

Ex. Evaluate $\arcsin\left(-\frac{1}{2}\right)$

What ∡ has a sine of $-\frac{1}{2}$?

$\arcsin\frac{1}{2} = \frac{\pi}{6}$

$\sin\frac{\pi}{6} = \frac{1}{2}$

$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

Ex. $\cos^{-1}(0)$ $\cos\frac{\pi}{2} = 0$

What ∡ has a cosine of 0

$\boxed{\cos^{-1}(0) = \frac{\pi}{2}}$

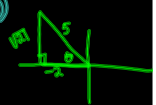
Ex. Evaluate $\sin\left(\arctan\frac{3}{4}\right)$

This is referring to an angle.

$\boxed{= \frac{3}{5}}$

Ex. Evaluate $\cot\left(\cos^{-1}\left(-\frac{2}{5}\right)\right)$

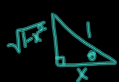
$\cot\theta = \frac{-2}{\sqrt{21}}$



Ex. Evaluate $\cos\left(\sin^{-1}\frac{x}{1}\right)$

$= \frac{\sqrt{1-x^2}}{1}$ OR $\sqrt{1-x^2}$

Ex. Evaluate $\tan(\arccos x)$



$\frac{\sqrt{1-x^2}}{x}$

Derivatives of inverse trig functions.

Figure out y' of $y = \arcsin x$

Means same as $\sin y = x$

Use implicit diff.

$y'\cos y = 1$

$y' = \frac{1}{\cos y}$

Need this in terms of just x.

Draw a picture: $y = \arcsin x$

$\sin y = x$

$\cos y = \sqrt{1-x^2}$

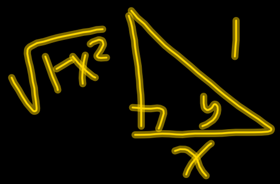
$y' = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$

Area A rectangular page is to contain 30 square inches of print. The margins on each side are 1 inch. Find the dimensions of the page such that the least amount of paper is used.

Find y' if $y = \arccos x$.

$$\cos y = x$$



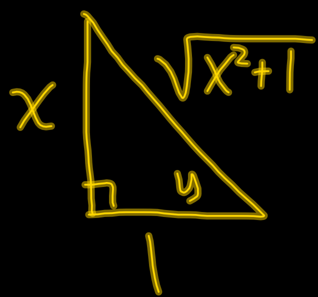
$$-y' \sin y = 1$$

$$y' = \frac{-1}{\sin y} \longrightarrow \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\arccos u\right] = \frac{-u'}{\sqrt{1-u^2}}$$

Figure out derivative of $y = \arctan x$.

$$\tan y = x$$



$$y' \cdot \sec^2 y = 1$$

$$y' = \frac{1}{\sec^2 y}$$

$$\sec y = \frac{\sqrt{x^2+1}}{1}$$

$$y' = \frac{1}{x^2+1}$$

$$\frac{d}{dx}\left[\arctan u\right] = \frac{u'}{u^2+1}$$

Flashcards w/ these 3 new deriv-rules

Ex. Find $f'(x)$ if $f(x) = \arcsin(2x)$

Rule $\frac{d}{dx}(\arcsin u) = \frac{u'}{\sqrt{1-u^2}}$

$u = 2x$

$$f'(x) = \frac{2}{\sqrt{1-4x^2}}$$

Ex. Find y' if $y = \left(\arccos \frac{x}{2}\right)^3$

$$y' = 3\left(\arccos \frac{x}{2}\right)^2 \cdot \left(\frac{-1/2}{\sqrt{1-\frac{x^2}{4}}}\right)$$

Ex. Find y' if $y = x\sin^{-1}(x) + \sqrt{(1-x^2)}^{1/2}$

$$y' = x \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}(x) + (-2x)\tfrac{1}{2}(1-x^2)^{-1/2}$$

$$y' = \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x) - \frac{x}{\sqrt{1-x^2}}$$

$$y' = \sin^{-1}(x)$$

Packet - Inv Trig functions

FR packet - close to finished

Be Prep book

The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \quad f'(0) = -4, \text{ and } f''(0) = 3.$$

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a. Show the work that leads to your answers.

(b) The function h is given by $h(x) = \cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.