AP Calculus AB
Thursday, November 7, 2013

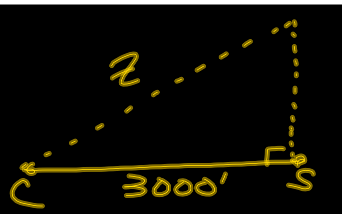
Historical Curves Assignment--do not let me forget about this!!!!

1. Witch of Agnesi – Gavin & Bryce
2.
3. Taylor & Emily Bifolium
4. Katie & Julie Folium of Descartes
5. Jake & Chuckie Newton's Trident

1. Find equation of tangent line at specific point.
2. Check your work with GGB.
3. Print out a graph with the original curve & the tangent line.
4. Paste onto colored paper (I have some).
5. Tape/paste your work onto the back of the paper.
6. I will be displaying these!

You have to do all five problems.
You will be assigned ONLY ONE to print & post.

A camera is mounted 3,000 feet from the space shuttle launching pad. The camera needs to pivot as the shuttle is launched and needs to keep the shuttle in focus. If the shuttle is rising vertically at 800 feet/sec when it is 4,000 feet high, how fast is the camera-to-shuttle distance changing?



$\frac{dy}{dt} = 800 \text{ ft/sec}$

$\frac{dz}{dt} = ?$ when $y = 4000'$

$z = 5000'$

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$$

$$\frac{dx}{dt} = 0 \quad (4000\text{ ft})\left(800\frac{\text{ft}}{\text{sec}}\right) = (5000\text{ ft})\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{4\cancel{000}\cdot 800}{5\cancel{000}}\ \frac{\text{ft}}{\text{sec}}$$

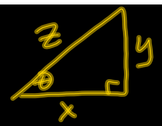
$$\boxed{\frac{dz}{dt} = 640\text{ ft/sec}}$$

In this problem, how fast is the angle of elevation of the camera changing at that moment in time?

What variable are we trying to find? ________ Since this is a function of θ, we need a trig function. There are three trig functions we could use. Let's try all three and determine which is best.

$\sin\theta = \frac{y}{z}$ $\qquad$ $\cos\theta = \frac{x}{z}$ $\qquad$ $\tan\theta = \frac{y}{x}$

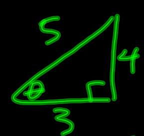
$\boxed{\frac{d\theta}{dt}}$



$$\frac{d\theta}{dt}\sec^2\theta = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$$

$$\frac{d\theta}{dt} = \frac{x\frac{dy}{dt}}{x^2\cdot\sec^2\theta}$$

$$\frac{d\theta}{dt} = \frac{dy/dt}{x\sec^2\theta}$$



$\sec\theta = \frac{5}{3}$

$$\frac{d\theta}{dt} = \frac{800\text{ ft/sec}}{(3000\text{ ft})(5/3)^2}$$

$$\frac{d\theta}{dt} = \frac{800}{3000\cdot\frac{25}{9}} = \frac{12}{125}\ \frac{\frac{\text{ft}}{\text{sec}}}{\text{ft}} \quad \frac{\cancel{\text{ft}}\,1}{\text{sec}\cdot\cancel{\text{ft}}} \quad 1/\text{sec}$$

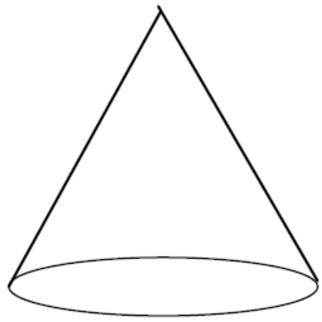
$$\frac{d\theta}{dt} = \frac{12}{125}\text{ Hz}$$

Sand is poured on a beach creating a cone whose radius is always equal to twice its height. If the sand is poured at the rate of 20 in^3/sec, How fast is the height changing at the time the height is a) 2 inches ?

$r = 2h$

$\frac{dV}{dt} = \frac{20 in^3}{sec}$

$\frac{dh}{dt} = ?$



$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{1}{3}\pi \left[r^2 \frac{dh}{dt} + h \cdot 2r \cdot \frac{dr}{dt} \right]$

$r = 2h \longrightarrow$ when $h = 2$, $r = 4$

$\frac{dr}{dt} = 2\frac{dh}{dt}$

Plugin

$20 = \frac{1}{3}\pi \left[4^2 \cdot \frac{dh}{dt} + 2 \cdot 2 \cdot 4 \cdot 2\frac{dh}{dt} \right]$

$20 = \frac{1}{3}\pi \left[16\frac{dh}{dt} + 32\frac{dh}{dt} \right]$

$20 = \frac{1}{3}\pi \cdot 48\frac{dh}{dt}$

$20 = \pi \cdot 16\frac{dh}{dt}$

$\frac{20}{16\pi} = \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{5}{(4\pi)}$ in/sec

A spherical Tootsie Roll Pop that you are enjoying is giving up volume at a steady rate of 0.25 in^3/min. How fast will the radius be decreasing when the Tootsie Roll Pop is .75 inches across?

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = -0.25 \frac{in^3}{min}$$

$$\frac{dr}{dt} = ? \text{ when } d = 0.75 \text{ in}$$

$$\frac{dV}{dt} = \frac{4}{3}\pi\left[3r^2 \frac{dr}{dt}\right]$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dV/dt}{4\pi r^2} = \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{-0.25}{4\pi\left(\frac{0.75}{2}\right)^2} \approx -0.14147 \frac{in}{min}$$

Write equation of tangent line @ $(6,-3)$ $x^2-y^2=27$

$2x\cdot\frac{dx}{dx}-2y\frac{dy}{dx}=0$

$2x-2y\frac{dy}{dx}=0$

$2x=2y\frac{dy}{dx}$

$\frac{x}{y}=\frac{dy}{dx}$

$\frac{6}{-3}=-2 \rightarrow$ slope @ $(6,-3)$

$y+3=-2(x-6)$

$$f(x)=\begin{cases} x^3-2x^2+3, & x\geq 1 \\ 3-x, & x<1 \end{cases}$$

① Det. if $f(x)$ is cont @ $x=1$.

$1^3-2\cdot1^2+3 \stackrel{?}{=} 3-1$

$1-2+3=3-1$

$\therefore f(x)$ is cont @ $x=1$.

② If yes, is $f(x)$ differentiable @ $x=1$.

$3x^2-4x$ $\qquad$ -1

$3\cdot1^2-4\cdot1$

-1 $\leftarrow$ slopes are same from left & right

Find $\frac{dy}{dx}$ $\quad (x+y)^2-4x=20y$

$2(x+y)^1[x'+y']-4x'=20y'$

$(2x+2y)(1+y')-4=20y'$

$2x+2xy'+2y+2y\cdot y'-4-20y'=0$

$y'(2x+2y-20)=4-2y-2x$

$y'=\frac{4-2y-2x}{2x+2y-20}$

$y'=\frac{2-y-x}{x+y-10}=\frac{-2+y+x}{-x-y+10}$

Find a & b to make $f(x)$ differentiabl

$$f(x)=\begin{cases} x^3+x+2, & x\geq 0 \\ a(x-1)-b, & x<0 \end{cases}$$

$3x^2+1=a$

$3\cdot0^2+1=a$

$a=1$

$0^3+0+2=a(-1)-b$

$2=-a-b$

$2=-1-b$

$b=-3$