AP Calculus AB
Wednesday, November 6, 2013

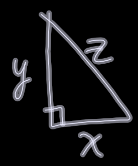
Check answers to yesterday's problem with someone.

1. Historical Curves Assignment
2.
3.
4.
5.

1. Find equation of tangent line at specific point.
2. Check your work with GGB.
3. Print out a graph with the original curve & the tangent line.
4. Paste onto colored paper (I have some).
5. Tape/paste your work onto the back of the paper.
6. I will be displaying these!

You have to do all five problems.
You will be assigned ONLY ONE to print & post.

A 13 foot ladder leans against a vertical wall. If the lower end of the ladder is pulled away at the rate 2 feet per second, how fast is the top of the ladder coming down the wall at a) the instant the top is 12 feet above the ground and b) 5 feet above the ground?




$$\frac{d}{dt}\left[x^2+y^2=z^2\right]$$

$$2x\cdot\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$$

$\frac{dy}{dt} \rightarrow$ Rate sliding vert.

$\frac{dx}{dt} \rightarrow$ rate sliding horiz.

$$\frac{dx}{dt}=2\frac{ft}{sec}$$

$$\frac{dy}{dt}=?$$

$$\frac{dz}{dt}=0$$

**When y = 12, find dy/dt.**

$$2x\frac{dx}{dt}+2y\boxed{\frac{dy}{dt}}=2z\frac{dz}{dt}$$


$$\frac{dx}{dt}=2\frac{ft}{sec} \quad \frac{dz}{dt}=0$$

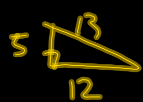
$$2x\left(2\frac{ft}{sec}\right)+2\cdot 12ft\cdot\frac{dy}{dt}=2\cdot z\cdot 0$$

$$4x\frac{ft}{sec}+24ft\,\frac{dy}{dt}=0$$

$$4\cdot 5\frac{ft^2}{sec}+24ft\cdot\frac{dy}{dt}=0$$

$$\frac{dy}{dt}=\frac{-20\,ft^2/sec}{24\,ft}=-\frac{5}{6}\frac{ft}{sec}$$

**When y = 5, find dy/dt.**

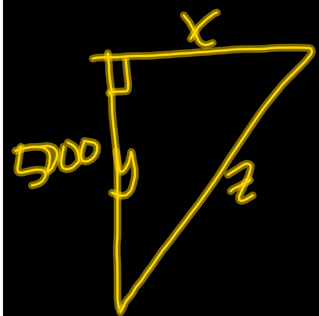



$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$$

$$2(12ft)\left(2\frac{ft}{sec}\right)+2(5ft)\frac{dy}{dt}=0$$

$$48\frac{ft^2}{sec}+10ft\cdot\frac{dy}{dt}=0$$

Example 1. An observer is tracking a small plane flying at an altitude of 5000 ft. The plane flies directly over the observer on a horizontal path at the fixed rate of 1000 ft/min. Find the rate of change of the distance from the plane to the observer when the plane has flown 12,000 feet after passing directly over the observer.




$x = 12000\,ft$

$z = 13000\,ft$

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$$

$\frac{dx}{dt} = 1000\,\frac{ft}{min}$

$\frac{dz}{dt} = ?$

$\frac{dy}{dt} = 0$

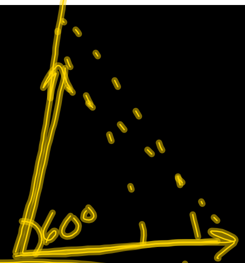
$$(12000\,ft)\left(1000\,\frac{ft}{min}\right) + 0 = (13000\,ft)\frac{dz}{dt}$$

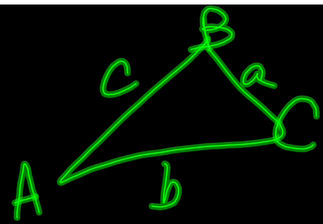
$$12000000\,\frac{ft^2}{min} = (13000\,ft)\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{12000\cancel{000}}{13\cancel{000}}\,\frac{ft}{min}$$

$$\boxed{\frac{dz}{dt} = \frac{12000}{13}\,ft/min}$$

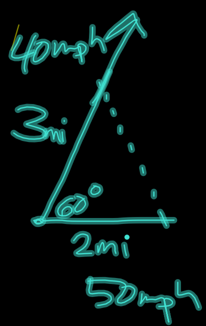
Example 8) Two cars are riding on roads that meet at a 60 degree angle. Car A is 3 miles from the intersection traveling at 40 mph and car B is 2 miles away from the intersection traveling at 50 mph. How fast are the two cars separating if a) they are both traveling away from the intersection and b) car A is traveling away from the intersection and car B is traveling towards it?

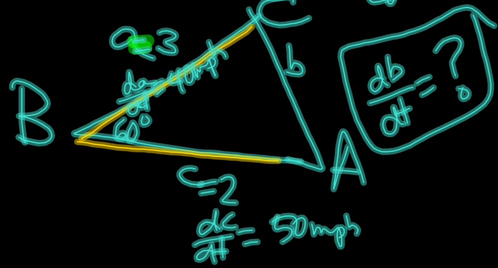



$\boxed{\text{Law of Cosines}}$

$$b^2 = a^2 + c^2 - \boxed{2ac}\boxed{\cos B}$$



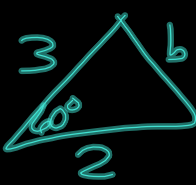




$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$2b\frac{db}{dt} = 2a\cdot\frac{da}{dt} + 2c\cdot\frac{dc}{dt} - 2\left[ac\cdot(-\sin B)\cdot\frac{dB}{dt} + (\cos B)\left(a\frac{dc}{dt} + c\cdot\frac{da}{dt}\right)\right]$$

$$2b\frac{db}{dt} = 2a\frac{da}{dt} + 2c\cdot\frac{dc}{dt} - 2\left[-ac\frac{dB}{dt}\sin B + \cos B\left(a\frac{dc}{dt} + c\cdot\frac{da}{dt}\right)\right]$$

Find b.




$$b^2 = 3^2 + 2^2 - 2\cdot 3\cdot 2\cos 60°$$

$$b^2 = 13 - 12\cdot\frac{1}{2}$$

$$b^2 = 7$$

$$b = \sqrt{7} \quad \cancel{-\sqrt{7}}$$

$$\frac{dB}{dt} = 0$$

$$2\sqrt{7}\cdot\frac{db}{dt} = 2\cdot 3\cdot 40 + 2\cdot 2\cdot 50 - 2\left[\cancel{-3\cdot 2\cdot 0\sin 60°} + \cos 60°(3\cdot 50 + 2\cdot 40)\right]$$

$$2\sqrt{7}\frac{db}{dt} = 240 + 200 - 2\left(\frac{1}{2}(230)\right)$$

$$2\sqrt{7}\frac{db}{dt} = 440 - 230$$

$$\frac{db}{dt} = \frac{105}{\sqrt{7}}\text{ mph}$$