AP Calculus AB
Tuesday, November 5, 2013

Bellwork:

Historical Curves Assignment

1.
2.
3.
4.
5.

1. Find equation of tangent line at specific point.
2. Check your work with GGB.
3. Print out a graph with the original curve & the tangent line.
4. Paste onto colored paper (I have some).
5. Tape/paste your work onto the back of the paper.
6. I will be displaying these!

You have to do all five problems.
You will be assigned ONLY ONE to print & post.

1. Find the value of a that makes the function continuous.

f. $f(x)=\begin{cases} a^2 - x^2, & x<2 \\ 1.5ax, & x \ge 2 \end{cases}$

$a^2 - 2^2 = 1.5a \cdot 2$

$a^2 - 4 = 3a$

$a^2 - 3a - 4 = 0$

$(a-4)(a+1) = 0$

$a=4 \quad a=-1$

If $a=4$

$f(x)=\begin{cases} 16-x^2, & x<2 \\ 6x, & x \ge 2 \end{cases}$

Check $a=-1$

$f(x)=\begin{cases} 1-x^2, & x<2 \\ -1.5x, & x \ge 2 \end{cases}$

Both work → $a=4$ & $a=-1$.

2. Bottom of page 60. Please do #7.

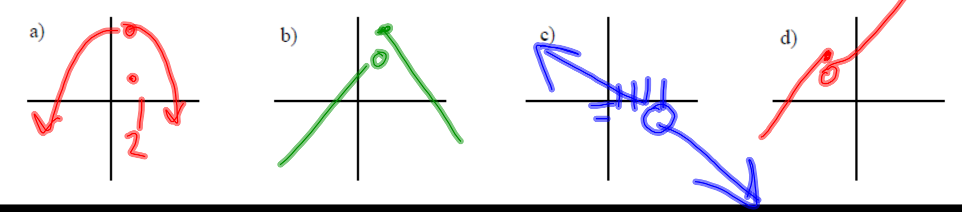
7. Sketch a function having the following attributes.

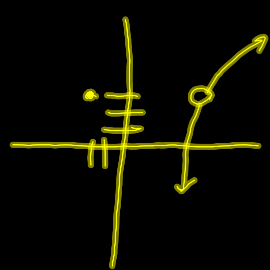
a) has a value of $f(2)$, a limit as x approaches 2, but is not continuous at $x = 2$.

b. has a step discontinuity at $x = 3$ where $f(3) = 7$

c. $\lim_{x \to 4} f(x) = -2$ but the function is not continuous at $x = 4$.

d. the value of $f(-2) = 3$ but there is no limit of $f(x)$ as x approaches -2 and no vertical asymptote there.





11. Find the values of a and b that make the function $f(x)$ differentiable.

a. $f(x)=\begin{cases} x^3, & x \ge 1 \\ a(x-2)^2+b, & x<1 \end{cases}$

b. $f(x)=\begin{cases} ax^2+10, & x \ge 2 \\ x^2-6x+b, & x<2 \end{cases}$

$1^{st} \to$ check/make Continuous!

$1^3 = a(1-2)^2 + b$

$1 = a+b$

$2^{nd} \to$ Differentiable

$3x^2 = 2a(x-2)$ when $x=1$

$3 \cdot 1^2 = 2a(1-2)$

$3 = -2a$

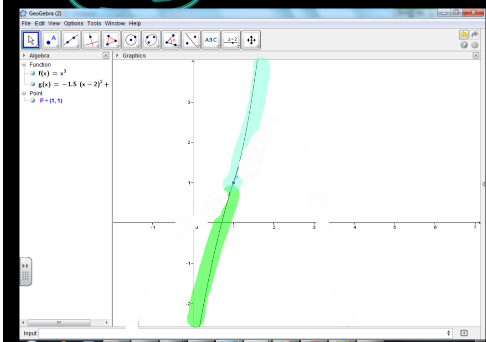
$a = -\frac{3}{2}$

$a+b = 1$

$-\frac{3}{2} + b = \frac{2}{2}$

$b = \frac{5}{2}$

$f(x) = \begin{cases} x^3, & x \ge 1 \\ -\frac{3}{2}(x-2)^2 + \frac{5}{2}, & x<1 \end{cases}$



11b

continuity

$a \cdot 2^2 + 10 = 2^2 - 6 \cdot 2 + b$

$4a + 10 = 4 - 12 + b$

$4a + 10 = -8 + b$

$4a + 18 = b$

$4(-\frac{1}{2}) + 18 = b$

$-2 + 18 = b$

$b = 16$

differentiability

$2ax = 2x - 6$

$2a \cdot 2 = 2 \cdot 2 - 6$

$4a = -2$

$a = -\frac{1}{2}$

$f(x) = \begin{cases} -\frac{1}{2}x^2 + 10, & x \ge 2 \\ x^2 - 6x + 16, & x<2 \end{cases}$

What comes 1^{st}, Continuity or Differentiability?

Example 1) Write the following statements mathematically.

a) John is growing at the rate of 3 inches/year.

b) My mutual fund is shrinking by 4 cents/day.

c) The radius of a circle is increasing by 4 ft/hr.

d) The volume of a cone is decreasing by 2 in^3/sec.

a) $\frac{dh}{dt} = \frac{3in}{yr}$ } The rate of change in height with respect to time.

b) $\frac{dm}{dt} = \frac{-4¢}{day}$

c) $\frac{dr}{dt} = \frac{4ft}{hr}$

d) $\frac{dV}{dt} = \frac{-2in^3}{sec}$

An oil tank spills oil that spreads in a circular pattern whose radius increases at the rate of 50 feet/min. How fast are both the circumference and area of the spill increasing when the radius of the spill is a) 20 feet and b) 50 feet?

Solution:

$A = \pi r^2$

$C = 2\pi r$

"how fast" → Rate

$\frac{dr}{dt} = \frac{50ft}{min}$

$\frac{dC}{dt} = ?$

$\frac{dA}{dt} = ?$

Find $\frac{dA}{dt}$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

@ $r = 20ft$

$\frac{dA}{dt} = 2\pi(20ft)\left(\frac{50ft}{min}\right)$

$\frac{dA}{dt} = 2000\pi \frac{ft^2}{min}$

@ $r = 50ft$

$\frac{dA}{dt} = 2\pi \cdot 50ft \cdot \frac{50ft}{min}$

$\frac{dA}{dt} = 5000\pi \frac{ft^2}{min}$

Find $\frac{dC}{dt}$.

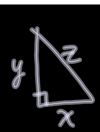
$C = 2\pi r$

$\frac{dC}{dt} = 2\pi \cdot \frac{dr}{dt}$

$\frac{dC}{dt} = 2\pi \cdot \frac{50ft}{min}$

$\boxed{\frac{dC}{dt} = 100\pi \frac{ft}{min}}$

) A 13 foot ladder leans against a vertical wall. If the lower end of the ladder is pulled away at the rate 2 feet per second, how fast is the top of the ladder coming down the wall at a) the instant the top is 12 feet above the ground and b) 5 feet above the ground?



$\frac{d}{dt}\left[x^2 + y^2 = z^2\right]$

$2x \cdot \frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$\frac{dy}{dt}$ → Rate sliding vert.

$\frac{dx}{dt}$ → rate sliding horiz.

$\frac{dx}{dt} = 2\frac{ft}{sec}$

$\frac{dy}{dt} = ?$

$\frac{dz}{dt} = 0$

When y = 12, find dy/dt.

When y = 5, find dy/dt.