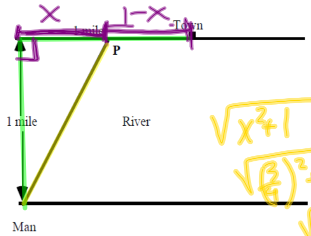
AP Calculus AB
Thursday, November 29, 2012

Bellwork...on the board
Go over review

Test Friday

6. A man is on the bank of a river that is 1 mile wide. He wants to travel to a town on the opposite bank but 1 mile upstream. He intends to row on a straight line to some point **P** on the opposite bank and then walk the remaining distance along the bank. How far from the town should he row in order to reach his destination in the shortest time if he can walk 5 mph and row 3 mph. (*10 points*)

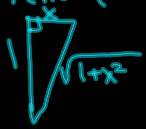


$\sqrt{x^2+1}$

$\sqrt{\left(\frac{3}{4}\right)^2+1}$

$\sqrt{\frac{9}{16}+\frac{16}{16}}$ $\quad \frac{5}{4}$

Yellow (water): $\sqrt{x^2+1}$ miles



Bank/land → $1-x$ miles

– need an eqn for time

Land: $5 \frac{mi}{hr}$

water: $3 mi/hr$

$d = rt$

$\frac{d}{r} = t$

$$Time = \frac{\sqrt{x^2+1}}{3} + \frac{1-x}{5}$$

$$t(x) = \frac{1}{3}(x^2+1)^{1/2} + \frac{1}{5} - \frac{1}{5}x$$

Take deriv. to find minimum.

$$t'(x) = \frac{1}{6}(x^2+1)^{-1/2} \cdot 2x - \frac{1}{5}$$

find critical values: $\frac{1}{3}(x^2+1)^{-1/2} \cdot x - \frac{1}{5} = 0$

$$\frac{x}{3\sqrt{x^2+1}} = \frac{1}{5}$$

$$(5x)^2 = \left(3\sqrt{x^2+1}\right)^2$$

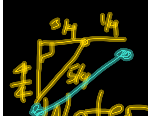
$$25x^2 = 9(x^2+1)$$

$$25x^2 = 9x^2 + 9$$

$$16x^2 = 9$$

$$x^2 = \frac{9}{16}$$

$$x = \frac{3}{4} \quad \left(\text{ignore } -\frac{3}{4}\right)$$



$\frac{d}{r}$

Water	time in water (3 mph)	land	time on land (5 mph)	total time
1	$\frac{1}{3}$	1	$\frac{1}{5}$	$\frac{8}{15}$
5/4	$\frac{5}{12}$ $\frac{25}{60}$	$\frac{1}{4}$	$\frac{1}{20}$ $\frac{3}{60}$	[illegible]
$\sqrt{2}$	$\frac{\sqrt{2}}{3}$	0	0	$\frac{5\sqrt{2}}{15}$

Find relative extrema and inflection points of $y = xe^{-x}$ Use calculus methods and be sure that your work is understandable.

Prod.

$y' = x(-e^{-x}) + 1 \cdot e^{-x}$

$y' = -xe^{-x} + e^{-x}$

set $y' = 0$

$-xe^{-x} + e^{-x} = 0$

$e^{-x}(-x+1) = 0$

$e^{-x} = 0$ no soln. $\quad -x+1=0$, $x=1$ critical value

y': $+$ | $-$ at 1

$x=2$: $-2e^{-2}+e^{-2}$, $\frac{-2}{e^2}+\frac{1}{e^2}$

Since y' goes from positive to negative @ $x=1$, y has a relative maximum @ $x=1$.

The relative maximum is $y = 1 \cdot e^{-1}$ $\frac{1}{e}$.

Point of inflection $y = 2e^{-2}$ $\left(2, \frac{2}{e^2}\right)$

$y' = -xe^{-x} + e^{-x}$

Points of infl $\to y''$

$y'' = (-x)(-e^{-x}) + (e^{-x})(-1) + (-e^{-x})$

$y'' = xe^{-x} - e^{-x} - e^{-x}$

$y'' = xe^{-x} - 2e^{-x}$

$xe^{-x} - 2e^{-x} = 0$

$e^{-x}(x-2) = 0$

$e^{-x} = 0$ no soln $\quad x-2=0$, $x=2$

$y'' = xe^{-x} - 2e^{-x}$: $-$ | $+$ at 2

$3e^{-3} - 2e^{-3}$, $\frac{3}{e^3} - \frac{2}{e^3}$

Since y'' changes signs @ $x=2$, y has a point of inflection @ $x=2$

$y = \ln(\ )$

Find y'

$= \ln(\cos x)$

$y' = \dfrac{(\ )'}{(\ )}$

$(\ )' = \dfrac{-\sin x}{\cos x}$
$= -\tan x$

$y' = \dfrac{-\tan x}{\ln(\cos x)}$

⑥ $y = \ln \sqrt[4]{\dfrac{x^2+1}{x^2-1}}$

$u = \sqrt[4]{\dfrac{x^2+1}{x^2-1}} = \left(\dfrac{x^2+1}{x^2-1}\right)^{1/4}$

$u' =$

use log prop. first.

$y = \ln\left(\dfrac{x^2+1}{x^2-1}\right)^{1/4}$

$y = \dfrac{1}{4}\ln\left(\dfrac{x^2+1}{x^2-1}\right)$

$y = \dfrac{1}{4}\left[\ln(x^2+1) - \ln(x^2-1)\right]$

Now, find y'.

$y' = \dfrac{1}{4}\left[\dfrac{2x}{x^2+1} - \dfrac{2x}{x^2-1}\right]$

$x^4 + e^{xy} - y^2 = 20 \qquad \dfrac{dy}{dx} = y'$

Implicit

$4x^3 + (x \cdot y' + y \cdot x) \cdot e^{xy} - 2y \cdot y' = 0$

$4x^3 + \underline{xy' \cdot e^{xy}} + ye^{xy} - \underline{2yy'} = 0$

$y'(xe^{xy} - 2y) = -ye^{xy} - 4x^3$

$y' = \dfrac{-ye^{xy} - 4x^3}{xe^{xy} - 2y}$