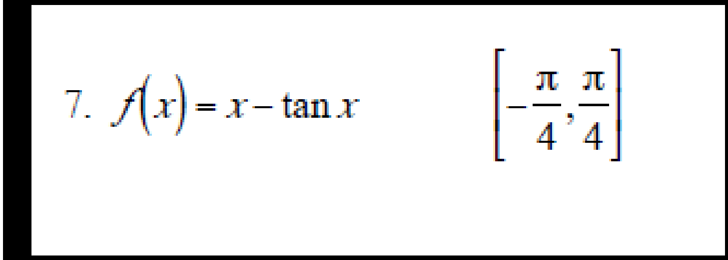
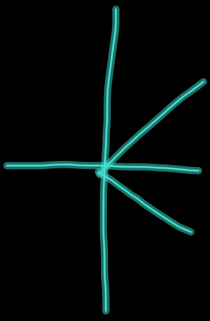
AP Calculus AB

Tuesday, November 26, 2013



7. $f(x) = x - \tan x$ $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$



$$f'(x) = 1 - \sec^2 x$$

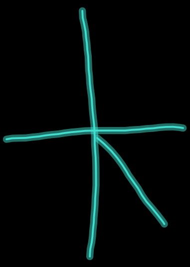
$$1 - \sec^2 x = 0$$

$$\sec^2 x = 1$$

$\sec x = 1$ $\sec x = -1$

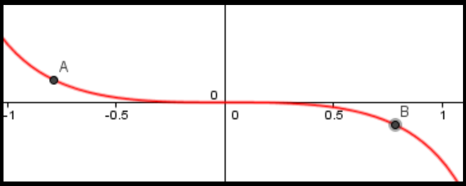
$\cos x = 1$ $\cos x = -1$

$x = 0$ not on $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$



x	$f(x) = x - \tan x$
$-\frac{\pi}{4}$	$-\frac{\pi}{4} - (-1) = 1 - \frac{\pi}{4} > 0$
0	$0 - \tan 0 = 0$
$\frac{\pi}{4}$	$\frac{\pi}{4} - \tan\frac{\pi}{4} = \frac{\pi}{4} - 1 < 0$

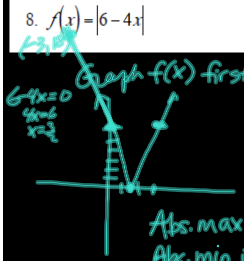
The absolute maximum is $1 - \frac{\pi}{4}$ & occurs at $x = -\frac{\pi}{4}$. The absolute minimum is $\frac{\pi}{4} - 1$ & occurs @ $x = \pi/4$.



8. $f(x) = |6 - 4x|$ [-3, 3]

Graph f(x) first bc f(x) is an abs.value

$6-4x=0$
$4x=6$
$x=\frac{3}{2}$



$$f(x)=\begin{cases} 6-4x, & x<1.5 \\ 4x-6, & x\geq 1.5 \end{cases}$$

Abs. max is 18 @ x=-3.
Abs. min is 0 @ x=1.5.

$g(x)=|u|$

$g'(x)=\frac{u\cdot u'}{|u|}$

9. What is the smallest possible slope to $y = x^3 - 3x^2 + 5x - 1$

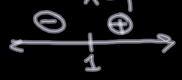
This is asking for the absolute minimum of the derivative (slope).

$\rightarrow y'=3x^2-6x+5$

$y''=6x-6$

$6x-6=0$

$x=1$



At x=1, the derivative reaches a minimum because the 2nd deriv goes from negative to positive. The minimum slope is 2. $(3(1)^2-6\cdot 1+5=2)$.

10. If a particle moves along a straight line according to $s(t) = t^4 - 4t^3 + 6t^2 - 20$, find

a) the maximum & minimum velocity on $0 \le t \le 3$.
b) the maximum & minimum acceleration on $0 \le t \le 3$

$v(t)=4t^3-12t^2+12t$

We need to find abs max & min of v(t).

$v'(t)=12t^2-24t+12$

$12(t^2-2t+1)=0$

$12(t-1)(t-1)=0$

$t=1$

t	$v(t)=4t^3-12t^2+12t$
0	0
1	4
3	$4\cdot 27-12\cdot 9+12\cdot 3=36$

The minimum velocity on $0\leq t\leq 3$ is 0 and occurs @ t=0. The maximum velocity is 36 and occurs @ t=3.

$v'(t)=12t^2-24t+12$

$a(t)=12t^2-24t+12$

Find abs max & min accel.

$a'(t)=24t-24$

$24t-24=0$

$t=1$ $12(t-1)^2$

t	$a(t)=12(t^2-2t+1)$
0	12
1	0
3	$12(4)=48$

The abs. max accel is 48 & occurs when t=3.
The abs min accel is 0 & occurs at t=1.

Find two positive numbers that minimize the sum of twice the first number plus the second if the product of the two numbers is 288.

Use calculus in your solution.

Let a & b be our two numbers

$ab = 288$

$b = 288/a$

$2a + b \rightarrow$ minimize

$f(a) = 2a + 288a^{-1}$

$f'(a) = 2 - 288a^{-2}$

$2 - 288a^{-2} = 0$

$\frac{288}{a^2} = \frac{2}{1}$

$2a^2 = 288$

$a^2 = 144$

$\frac{288}{169}$ $a = 12, \cancel{-12}$ "positive numbers"

Verify/check that $a = 12$ produces a min

$2 - \frac{288}{a^2}$ $\xleftrightarrow{\; - \;|\; + \;}$ 12

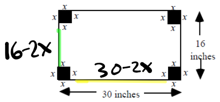
Since $f'(a)$ goes from $-$ to $+$ @ 12, $f(a)$ has a relative min @ $a = 12$.

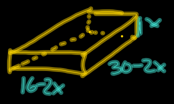
$b = \frac{288}{12} = 24$

The two numbers are 12 & 24.

Example 4) An open box is to be made from a piece of metal 16 by 30 inches by cutting out squares of equal size from the corners and bending up the sides. What size square should be cut out to create a box with greatest volume. What is the maximum volume as well?

Primary





$V = x(16-2x)(30-2x)$

$V = 4x^3 - 92x^2 + 480x$

$V' = 12x^2 - 184x + 480$

$V' = 0$

$0 = 4(3x^2 - 46x + 120)$

$0 = 4(3x - 10)(x - 12)$

$3 \cdot 11^2 - 46 \cdot 11 + 120$ $x = \frac{10}{3}$ $x = 12$

$363 - 506 + 120$

$+ \;|\; - \;|\; +$

$\frac{10}{3}$ 12

at $x = \frac{10}{3}$ the volume is maximized because the derivative goes from positive to negative.

$V = \frac{10}{3}\left(16 - 2\left(\frac{10}{3}\right)\right)\left(30 - 2 \cdot \frac{10}{3}\right)$

$V = \frac{10}{3}\left(\frac{28}{3}\right)\left(\frac{70}{3}\right)$

$\boxed{725.926 \text{ in}^3}$

- optimization problems MMM
- Packet of FRQ