AP Calculus AB
Monday, November 25, 2013

ETEH in here today & tomorrow..

$f(x)=\sqrt{1-x^2}$

Find $f'(x)$

$f(x)=(1-x^2)^{1/2}$

$f'(x)=\frac{1}{2}(1-x^2)^{-1/2}(-2x)$

$f'(x)=\frac{-2x}{2\sqrt{1-x^2}}$

$f'(x)=\frac{-x}{\sqrt{1-x^2}}$

$\frac{-x}{\sqrt{1-x^2}}=-1$

$\pm\sqrt{\frac{1}{2}}$
$\pm 0.7071..$

$-x=-1\sqrt{1-x^2}$

$x^2=\sqrt{1-x^2}^{\,2}$

$x^2=1-x^2$

$2x^2=1$

$x^2=\frac{1}{2}$

$x=\pm\sqrt{1/2}$

(6 points) 2. Find all the values of c on the closed interval $[0,1]$ that satisfy the Mean Value Theorem for $f(x)=\sqrt{1-x^2}$, or explain why no such values exist.

1st check for C & D

$f(x)=\sqrt{1-x^2}$

$1-x^2\geq 0$

$1\geq x^2$

$x^2\leq 1$

$-1\leq x\leq 1$

$[0,1]$ $f(x)$ is cont. on $[0,1]$ and diff on $(0,1)$. $\therefore$ we can apply MVT.

$m_{sec}=\frac{f(1)-f(0)}{1-0}$

$m_{sec}=\frac{\sqrt{1-1^2}-\sqrt{1-0^2}}{1}=-1$

$f'(x)=-1$ & solve (we did this above.)

$x=\pm\sqrt{\frac{1}{2}}$ only $\sqrt{\frac{1}{2}}$ is on $[0,1]$.

$f(x)=3x^2-24x-1 \quad [-1,5]$

$f'(x)=6x-24$

$6x-24=0$

$x=4$

	x	$f(x)=3x^2-24x-1$
endpt	-1	$3\cdot1+24-1=26$
critical value	4	$3\cdot16-96-1=-49$
endpt	5	$3\cdot25-24\cdot5-1=-46$

The absolute maximum on $[-1,5]$ is 26 and occurs when $x=-1$.

The absolute minimum is -49 and occurs when x = 4.

© $f(x)=3x^{2/3}-2x+1 \quad [-1,8]$

$f'(x)=2x^{-1/3}-2$

$2x^{-1/3}-2=0$

$x^{-1/3}=1$

$\frac{1}{\sqrt[3]{x}}=1$

$x=1$

CV also occur where $f'(x)$ is undefined.

$x=0$

	x	$f(x)=3x^{2/3}-2x+1$
ep	-1	$3(-1)^{2/3}-2(-1)+1=6$
cv	0	1
cv	1	2
ep	8	$3\cdot8^{2/3}-2\cdot8+1=-3$

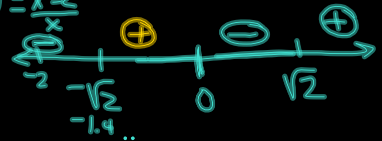
The absolute maximum 6 & occurs when $x=-1$.

The abs. min. is -3 & occurs when $x=8$.

② $h'(x) = \dfrac{x^2-2}{x}$ $\quad x \neq 0 \quad$ $h(4) = -3$

(a)
$h'(x) = 0$
$x^2 - 2 = 0$ $\quad$ $x = 0$ is still a critical value
$x = \pm\sqrt{2}$

$h'(x) = \dfrac{x^2-2}{x}$



h has a local minimum @ $x = -\sqrt{2}$ because $h'(x)$ goes from negative to positive @ $x = -\sqrt{2}$.
h has another local minimum @ $x = \sqrt{2}$ bc $h'(x)$ goes from negative to positive @ $x = \sqrt{2}$.

h does not have a local maximum at x=0 because h(x) is not defined at x=0.

(b) $h'(x) = \dfrac{x^2-2}{x}$

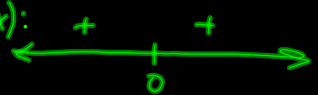
$$h''(x) = \frac{x(2x) - (x^2-2)(1)}{x^2}$$

$$h''(x) = \frac{2x^2 - x^2 + 2}{x^2}$$

$$h''(x) = \frac{x^2+2}{x^2}$$

$h''(x) = 0$?
$x^2 + 2 \neq 0$
$x^2 \neq 0 \to x = 0$

$f''(x)$:



Since $h''(x)$ is always > 0 (where it exists), $h(x)$ is always concave up.

(c) $h(4) = -3$ $\qquad$ $h'(x) = \dfrac{x^2-2}{x}$

$h'(4) = \dfrac{16-2}{4} = \dfrac{7}{2}$

$y + 3 = \frac{7}{2}(x - 4)$

(d)  Since $h(x)$ is concave up, the tangent line will lie below the curve.

d. $f(x) = \sin^2 x + \cos x \quad [0, 2\pi]$

$f'(x) = 2\sin x \cdot \cos x - \sin x$

$0 = 2\sin x \cdot \cos x - \sin x$

$0 = (2\cos x - 1)\sin x$

$2\cos x - 1 = 0 \qquad \sin x = 0$

$\cos x = \frac{1}{2} \qquad 0, \pi, 2\pi$

$\frac{\pi}{3}, \frac{5\pi}{3}$

x	$f(x) = \sin^2 x + \cos x$
0	1
$\frac{\pi}{3}$	$\frac{5}{4}$
π	-1
$\frac{5\pi}{3}$	$\frac{5}{4}$
2π	1

The absolute minimum occurs @ $x = \pi$ and is -1.

The absolute maxima occur @ $x = \frac{\pi}{3}$ & $x = \frac{5\pi}{3}$ and are $\frac{5}{4}$.