AP Calculus AB

Tuesday, November 20, 2012

1. Find f'(x) using the DEFINITION OF THE DERIVATIVE:

$f(x) = x^2 - x - 1$

2. Evaluate the limit using a graphing utility:

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

$f(x)=x^2-x-1$

① $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$f'(x)=\lim_{h\to 0}\frac{(x+h)^2-(x+h)-1-[x^2-x-1]}{h}$

$f'(x)=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2\cancel{-x}-h\cancel{-1}\cancel{-x^2}\cancel{+x}\cancel{+1}}{h}$

$f'(x)=\lim_{h\to 0}\frac{2xh+h^2-h}{h}$

$f'(x)=\lim_{h\to 0} 2x-1+h$

$\boxed{f'(x)=2x-1}$

What is the derivative of e^x?

Let $f(x)=e^x$ & use defn of deriv.

$f'(x)=\lim_{h\to 0}\frac{e^{x+h}-e^x}{h}$

$f'(x)=\lim_{h\to 0}\frac{e^x\cdot e^h-e^x}{h}$

$f'(x)=\lim_{h\to 0}\frac{e^x(e^h-1)}{h}$

$f'(x)=e^x\cdot\boxed{\lim_{h\to 0}\frac{e^h-1}{h}}$ $=1$

$f'(x)=e^x$

Ex. If $f(x)=e^{2x}$, find $f'(x)$.

$f'(x)=2e^{2x}$

$\frac{d}{dx}[e^u]=u'\cdot e^u$

Ex. $f(x)=e^{2x^2}$

$f'(x)=4xe^{2x^2}$

$x^{1/2}$

Ex. $f(x)=e^{\sin x}$

$f'(x)=\cos x\cdot e^{\sin x}$

$g(x)=e^{\sqrt{x}}$

$g'(x)=\frac{1}{2}x^{-1/2}\cdot e^{\sqrt{x}}$

$g'(x)=\frac{e^{\sqrt{x}}}{2\sqrt{x}}$

Derive?

$y=\ln x$ } equivalent forms

$e^y=x$ }

$2^3=8 \Leftrightarrow \log_2 8=3$

Take deriv wrt/x.

$\frac{dy}{dx}e^y=1$

Solve for $\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{1}{e^y}$

$e^y=x$

So.....

$\frac{dy}{dx}=\frac{1}{x}$ $\therefore$ If $y=\ln x$, then $y'=\frac{1}{x}$.

$\boxed{\frac{d}{dx}[\ln u]=\frac{u'}{u}}$

Ex. $f(x)=\ln x^2$, then $f'(x)=\frac{2x}{x^2}$

$f'(x)=\frac{2}{x}$

$f(x)=\ln(\sin x)$

$f'(x)=\frac{\cos x}{\sin x}$

$f'(x)=\cot x$

$f(x)=\ln(\tan x)$

$f'(x)=\frac{\sec^2 x}{\tan x}$

$f'(x)=\frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}}$

$=\frac{1}{\cos^2 x}\cdot\frac{\cos x}{\sin x}$

$=\frac{1}{\cos x}\cdot\frac{1}{\sin x}$

$=\csc x\cdot\sec x$

Calculus Differentiation Rules Quiz   Name: ______________________

Fill in the blanks. *c* is a constant, *u* and *v* are functions, and *x* is a variable. n is a constant.

$\frac{d}{dx}[cu] = c \cdot u'$	$\frac{d}{dx}[u \pm v] = u' \pm v'$
$\frac{d}{dx}[uv] = uv' + v \cdot u'$	$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$ Quotient Rule
$\frac{d}{dx}[c] = 0$	$\frac{d}{dx}[u^n] = n \cdot u^{n-1} \cdot u'$ Power Rule
$\frac{d}{dx}[x] = 1$	$\cancel{\frac{d}{dx}[\lvert u \rvert] =}$
$\frac{d}{dx}[\ln u] = \frac{u'}{u}$	$\frac{d}{dx}[e^u] = u'e^u$
$\frac{d}{dx}[\sin u] = u' \cos u$	$\frac{d}{dx}[\cos u] = -u' \sin u$
$\frac{d}{dx}[\tan u] = u' \sec^2 u$	$\frac{d}{dx}[\cot u] = -u' \csc^2 u$

$$f(x) = e^2$$

$$f'(x) = 0$$

Power Rule → constant

$$f(x) = x^3$$

$$f(x) = e^{x^2}$$

→ variable

$$f'(x) = 2xe^{x^2}$$

$$f(1) = e^{1^2}$$

$$\frac{d}{dx}[\sec u] = u' \sec u \tan u$$

$$\frac{d}{dx}[\csc u] = -u' \csc u \cot u$$

$$\frac{d}{dx}[\cot u] = -u' \csc^2 u$$

page 322: 45-59 odd (ln)

page 348: 39-61 odd (e^x)

47

Find the derivative of the function

$y = (\ln x)^4$.

$y' = 4(\ln x)^3 \cdot \frac{1}{x}$

$y' = \frac{4(\ln x)^3}{x}$

49.

Find the derivative of the function

$y = \ln\left(x\sqrt{x^2 - 1}\right)$.

Could do this:

$u = x\underbrace{(x^2-1)^{1/2}}_{\text{product/chain rule}}$

$y = \ln u$

OR

$y = \ln x + \ln (x^2-1)^{1/2}$

Using this log property allows us to avoid using the product rule.

$y = \ln x + \ln(x^2-1)^{1/2}$

$y = \ln x + \frac{1}{2}\ln(x^2-1)$

now calculus....

$y' = \frac{1}{x} + \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}x}{x^2-1}$

$y' = \frac{1}{x} + \frac{x}{x^2-1}$

$y' = \frac{x^2-1+x^2}{x(x^2-1)}$

$y' = \frac{2x^2-1}{x(x^2-1)}$

Find the derivative of the function

$y = \ln\left(x\sqrt{x^2 - 1}\right)$.

1. $y = \ln x\sqrt{x^2 - 1}$
2. $\quad = \ln x + \frac{1}{2}\ln(x^2 - 1)$
3. $\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2}\left(\frac{2x}{x^2 - 1}\right)$
4. $\quad = \frac{2x^2 - 1}{x(x^2 - 1)}$