

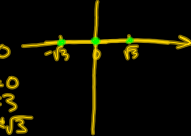
Example 2) Find all values between the intercepts for which Rolle's theorem holds for the roots of $f(x) = 3x^3 - x^4$. Confirm using your calculator.

$$3x^2 - x^4 = 0$$

$$x^2(3 - x^2) = 0$$

$$x = 0, \quad 3 - x^2 = 0$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$


$$f'(x) = 6x - 4x^3$$

$$6x - 4x^3 = 0 \quad \text{The slope between } (-\sqrt{3}, 0) \text{ and } (\sqrt{3}, 0) \text{ is 0.}$$

$$2x(3 - 2x^2) = 0$$


$$2x = 0 \quad 3 - 2x^2 = 0$$

$$x = 0 \quad 2x^2 = 3$$

$$x^2 = \frac{3}{2}$$

$$x = \pm\sqrt{\frac{3}{2}}$$

You are driving in a car traveling at 50 mph and you pass a police car. Four minutes later, you pass a second police car and you are traveling at 50 mph. The distance between the two police cars is five miles. The second police car is looking for speeding. How can he prove that you were speeding?



$$\frac{5 \text{ mi}}{4 \text{ min}} = \frac{5 \text{ mi}}{\frac{1}{15} \text{ hr}} = 75 \text{ mph}$$

$$\frac{50 \text{ mi}}{\text{hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = \frac{5}{6} \frac{\text{mi}}{\text{min}} = \frac{5 \text{ mi}}{6 \text{ min}}$$

Example 3) Given $f(x) = 3 - \frac{5}{x}$. Find the value of c in the interval $(1, 5)$ that satisfies the mean value theorem.

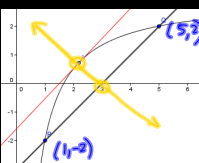


To the left is the graph of the function.

Visually, what is the mean value theorem trying to find? Draw it in.

$$f(1) = 3 - \frac{5}{1} = -2$$

$$f(5) = 3 - \frac{5}{5} = 2$$



equation of secant

$$m = \frac{2 - (-2)}{5 - 1} = 1$$

$$y - 2 = 1(x - 5)$$

$$y - 2 = x - 5$$

$$y = x - 3$$

x-int: (3, 0)

$$m_{\perp} = -1$$

$$y - 0 = -1(x - 3)$$

$$y = -x + 3$$

Where does $y = -x + 3$ intersect $f(x)$?

$$-x + 3 = 3 - \frac{5}{x}$$

$$-x = -\frac{5}{x}$$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

How can we check this? We need to find equation of tangent line @ $x = \sqrt{5}$.

$$f(x) = 3 - 5x^{-1}$$

$$f'(x) = \frac{5}{x^2}$$

$$f'(\sqrt{5}) = \frac{5}{(\sqrt{5})^2} = 1$$

Will this work all of the time?

MVT:

slope of secant line $\rightarrow 1$

$$\frac{5}{x^2} = 1$$