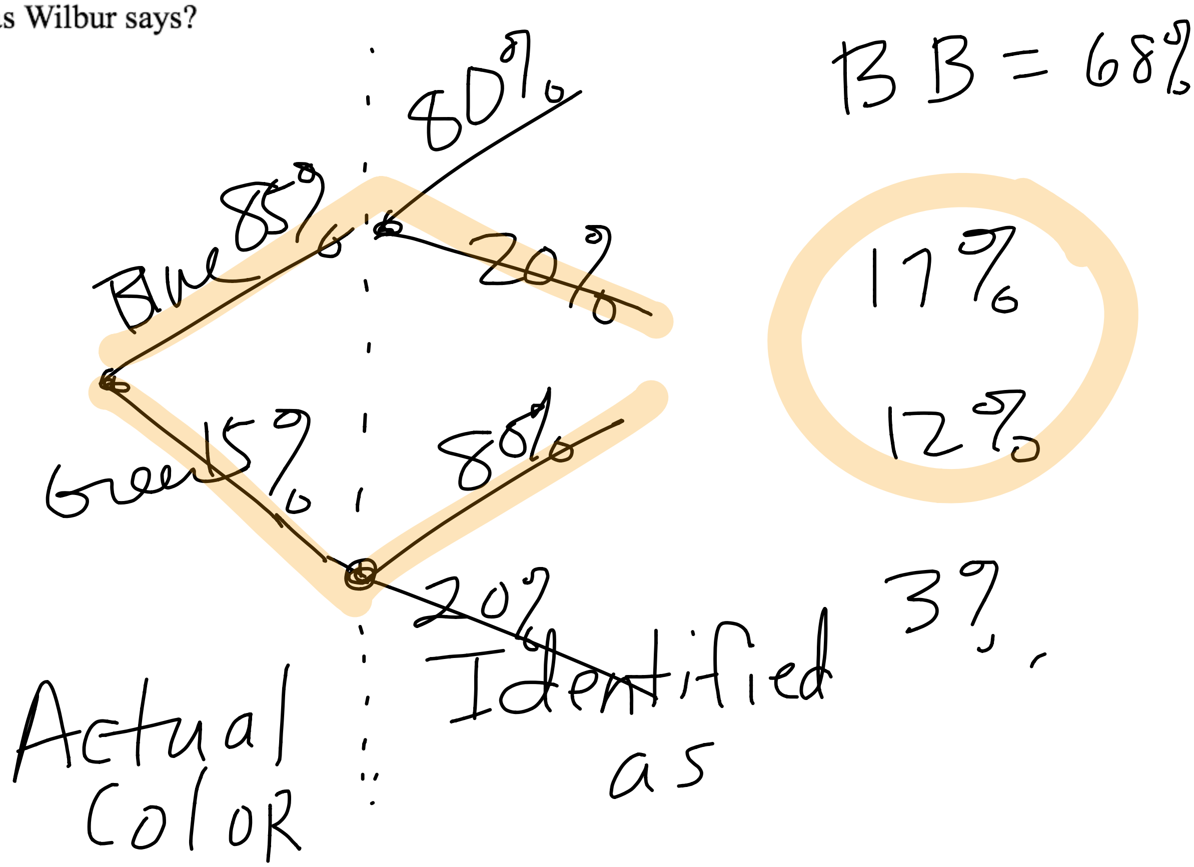
Problem 2: Exactly two cab companies operate in Belleville. The Blue Company has blue cabs, and the Green Company has Green Cabs. Exactly 85% of the cabs are blue and the other 15% are green. A cab was involved in a hit-and-run accident at night. A witness, Wilbur, identified the cab as a Green cab. Careful tests were done to ascertain peoples' ability to distinguish between blue and green cabs at night. The tests showed that people were able to identify the color correctly 80% of the time, but they were wrong 20% of the time. What is the probability that the cab involved in the accident was indeed a green cab, as Wilbur says?



$$\frac{12}{29} = 41.3\%$$

Two events are mutually exclusive (disjoint) if they cannot occur at the same time. (OR)

Two events are independent if the occurrence of one does not change the probability of the other occurring.

First \ Second	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$\frac{2}{11}$

$$\frac{P(A \cup B)}{P(B)} \quad \frac{\frac{2}{\not{36}}}{\frac{6}{\not{36}}} \rightarrow \frac{2}{6}$$

$$\frac{\frac{2}{36}}{\frac{11}{36}} = \frac{2}{11}$$

A = at least one dice is 4

B = sum of 7

Probability Terms and Rules

You need to know all these terms and rules and be able to apply then. Your book has a good source of exercises and this packet supplements that.

Probability – the study of random phenomena. The probability of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions.

The probability of an event is a number from 0 to 1 inclusive.

Probability 0	Probability 0.5	Probability 1
It cannot happen	As likely to happen as not	It must happen

Sample Space - the set of all possible outcomes of an experiment with random outcome

Example: Experiment: rolling a die: Sample space = $\{1,2,3,4,5,6\}$

Since the sample space is everything that can occur, adding the probabilities of everything in the sample Space must add up to 1.

Event: an outcome or outcomes in a sample space:

Example: Experiment: rolling a die. Event: result is even.

Probability of an event: $\dfrac{\text{number of ways the event can occur}}{\text{number of members in the sample space}}$

Example: Event: rolling a die. Probability (rolling an even number) = $\dfrac{3}{6} = \dfrac{1}{2}$

Multiplication principal: If you can do a task in m ways and a second task in n ways, then the number of ways that both tasks can be done is $a \times b$ ways

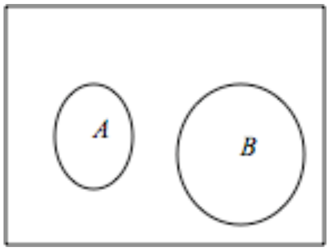
Example: Events: Rolling a die then tossing a coin. The number of elements in the sample space is 6(2) = 12.

Complement of an event: The complement of even A is the event that A does not occur:

Formula: $P\left(A^C\right) = 1 - P(A)$

Example: Experiment: Rolling a die: A: rolling a number < 3. $P(A) = \dfrac{2}{6}$ so $P\left(A^C\right) = 1 - P(A) = \dfrac{4}{6} = \dfrac{2}{3}$

Disjoint events: Two events A and B are disjoint (sometimes called **mutually exclusive)** if they have no outcomes in common and can never happen simultaneously. This can be shown in the **Venn Diagram** below. Notice that A and B have nothing in common.



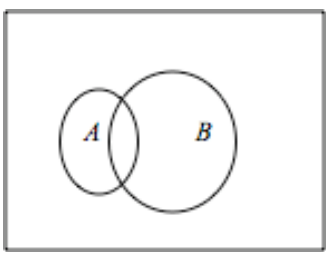
Formula: $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$

Example: A cooler contains 20 bottles made up of 8 Cokes, 5 Pepsis and 7 waters. The Probability of choosing a Coke or Pepsi is $\frac{8}{20} + \frac{5}{20} = \frac{13}{20}$.

General Addition rule. If two (or more) events are not disjoint the above formula doesn't work. The rule is:

Formula: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

The Venn Diagram below shows this relationship: If we simply add the probabilities, we will be adding the middle section twice. So we have to subtract one of them.



Example: In a class, there are 12 boys made up of 8 Seniors and 4 Juniors . There are also 8 girls, made up of 3 Seniors and 5 Juniors. Find the probability of choosing a boy or a Senior.

Note that choosing a boy and choosing a Senior are not disjoint (they can occur simultaneously). So the Probability of choosing a boy or a senior = P(Boy) + P(Senior) – P(Senior boy) = $\frac{12}{20} + \frac{11}{20} - \frac{8}{20} = \frac{15}{20} = \frac{3}{4}$.

A chart best shows this relationship:

	Boy	Girl	Total
Senior	8	3	11
Junior	4	5	9
Total	12	8	

Independence: Two events are independent if knowing one occurs doesn't change the probability of the other occurring. This is the non-mathematical definition.

Example: Tossing one coin and then another coin are independent events. The result of the second coin toss has nothing to do with the results of the first coin toss.

Proving independence is difficult. For instance is making a second foul shot in basketball independent of the first foul shot? Arguments?

Mathematical way of proving independence: If two events are independent then $P(A)\cdot P(B)=P(A \text{ and } B)$. Also, if $P(A)\cdot P(B)=P(A \text{ and } B)$, then the two events are independent: This is called the mathematical rule. If events are independent, to find the probability of them all happening, you may multiply the probabilities.

Example: In the problem above, are choosing a Senior and choosing a boy independent?

If so, $P(\text{Senior})\cdot P(\text{Boy})=P(\text{Senior boy})$: $\frac{11}{20}\cdot\frac{12}{20}=\frac{8}{20}$? $.33\neq .40$ so events are not independent

Choosing a boy influences the probability of choosing a Senior.

Conditional Probability: The probability of one event happening, given that another event has happened.

The way this is written is $P(B\mid A)$ which means the probability of B occurring given that A occurred.

Again, let's use this example:

	Boy	Girl	Total
Senior	8	3	11
Junior	4	5	9
Total	12	8	20

To find the $P(\text{Boy}\mid\text{Senior})$ (the probability of choosing a boy given that we chose a senior), we look at the chart and see that there are 11 seniors and 8 of them are boys. So $P(\text{Boy}\mid\text{Senior})=\frac{8}{11}$

To find the $P(\text{Senior}\mid\text{Boy})$ (the probability of choosing a Senior given that we chose a boy), We look at the chart and see that there are 12 boys and 8 of them are Seniors. So $P(\text{Senior}\mid\text{Boy})=\frac{8}{12}=\frac{2}{3}$

The formula for $P(B\mid A)=\frac{P(A \text{ and } B)}{P(A)}$. Using this formula:

$$P(\text{Boy}\mid\text{Senior})=\frac{P(\text{Senior Boy})}{P(\text{Senior})}=\frac{\frac{8}{20}}{\frac{11}{20}}=\frac{8}{11}$$

$$P(\text{Senior}\mid\text{Boy})=\frac{P(\text{Senior Boy})}{P(\text{Boy})}=\frac{\frac{8}{20}}{\frac{12}{20}}=\frac{8}{12}=\frac{2}{3}$$

Problems using the Addition Rule

1. A bag contains marbles: 10 red, 12 green, 15, blue, 8 red, 12 white, and 7 black. A marble is chosen. Find the probabilities of choosing:

a) red

b. black or white

c) red or green

d) red, green or blue

e) not white

f) not white or black

2. Jerrold's TV can pick up 120 stations. At 8 PM, 42 stations are showing commercials, 18 sports, 25 movies, 7 news shows, 13 comedy programs, 12 drama programs, and 3 are sales program (like QVC). A station is chosen at random: Find the following probabilities:

a) choosing a news show

b) choosing a commercial

c) a comedy or drama

d) sports, movie or news

e) not a sales program

f) neither sports or a movie

3. A pet store has 25 pets, 16 dogs and 9 cats. Of the dogs, 13 are puppies and of the cats 6 are kittens. A pet is chosen at random. Find the following probabilities:

a) a dog

b) a cat

c) a puppy

d) a kitten

e) a puppy or a kitten

d) neither a puppy or kitten

4. A rental car lot has 49 American made cars and 26 Foreign cars. Of the American cars, 35 of them are white and of the foreign cars, 15 are white. A car is chosen at random. Find the probabilities (make a chart).

a) American

b) foreign

c) American or foreign

d) not white

e) American or white

f) Foreign or not white

5. Marci's Mom put a bunch of cookies in a cookie jar. There were 27 chocolate cookies and 32 vanilla cookies. Of the chocolate cookies, 7 of them were generic. Of the vanilla cookies, 10 of them were generic. A cookie is chosen at random. Find the probability of choosing. Make a chart.

a) chocolate

b) vanilla

c) chocolate non-generic

d) chocolate or non generic

e) vanilla generic

f) vanilla or generic

6. A pizza shop has two sizes of pizzas, large and small. On a certain day, a pizza shop made 59 plain pizzas and 72 pizzas with toppings. Of the 59 plain pizzas, 19 were small and of the 72 pizzas with toppings, 42 were large. A pizza is chosen at random. Find the following probabilities. Make a chart.

a) large

b) small

c) with toppings

d) plain

e) large plain

f) small with toppings

g) small or plain

h) large or with toppings

i) small or large

7. Over a 5 year period of time, 80% of winter days in Philadelphia had an average temperature above freezing. 20% of those days had precipitation. Of the days with an average temperature below freezing, 15% had precipitation. A winter day is chosen at random. Find the probability of choosing a day: Make a chart.

a) above freezing

b) with precipitation

c) freezing with precipitation

d) freezing or precipitation

e) above freezing with no precipitation

f) above freezing or no precipitation

8. A computer store stocks the computers according to the following chart:

	Dell	HP	Mac
Laptop	20	25	12
Desktop	19	32	8

A computer is chosen at random. Find the probability of choosing

a) a Mac

b) a Dell or HP

c) a laptop

d) a Mac laptop

e) a Mac or laptop

d) a Dell or desktop

9. A classroom with 30 students contains White students, Asian Students, and African-American students. These students have either black hair, brown hair, or blond hair. Following is a chart, partially filled in. First complete the chart. A student is chosen at random. You are given two events with a connector (or / and). Determine if the two events are disjoint and then find the probability of the event.

	White	Af - Amer	Asian	Total
Black	1	8		
Brown	7			14
Blond	2	1	1	
Total		11		30

a) a white student or African American student.

b) a student without blond hair.

c) an Asian student with brown hair.

d) a student who is either Asian or had brown hair.

e) A student who is either African American or has black hair.

f) a student who is either African American or Asian or has black hair.

Determine which events are disjoint:

g) White and Asian

h) Brown and Blond

i) Not White and not blond