AP Calculus AB
Thursday, November 1, 2012

Test Tuesday:
Topics:
- Implicit differentiation
- Related Rates
- Straight Line Motion
- Continuity & Differentiability

Bellwork:

If a function is continuous everywhere, is it differentiable everywhere?

If a function is differentiable everywhere, is it continuous everywhere?

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In dealing with continuity of a piecewise function, we need to examine the x-value where the rule changes.

Example 3) $f(x)=\begin{cases} x^2-3, x\ge 1 \\ 1-x, x<1 \end{cases}$

$f(1)=-2$

not same

$1-1=0$

$f(x)$ is discontinuous @ $x=1$.

Example 4) $f(x)=\begin{cases} x^2+3x-2, x\ge -2 \\ -x^2, x<-2 \end{cases}$

Example 5) $f(x)=\begin{cases} 3^{-x}-1, x\ge -1 \\ \dfrac{1}{x+1}, x<-1 \end{cases}$

Example 6) $f(x)=\begin{cases} \dfrac{x-4}{x^2-16}, x\ne 4 \\ \dfrac{1}{3x-4}, x=4 \end{cases}$

Find the value of the constant k that will make the function continuous. Verify by calculator.

Example 7) $f(x)=\begin{cases} 3x+2, x\ge 1 \\ \ldots \end{cases}$

Example 8) $f(x)=\begin{cases} kx^2, x\ge 2 \\ \ldots \end{cases}$

Example 9) $f(x)=\begin{cases} k^2-12x, x\ge 1 \\ \ldots \end{cases}$

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Find the value of the constant k that will make the function continuous. Verify by calculator.

Example 7) $f(x)=\begin{cases}3x+2, x\ge 1\\ 2k-x, x<1\end{cases}$ Example 8) $f(x)=\begin{cases}kx^2, x\ge 2\\ kx-6, x<2\end{cases}$ Example 9) $f(x)=\begin{cases}k^2-12x, x\ge 1\\ kx, x<1\end{cases}$

$3(1)+2=2k-1$

$5=2k-1$

$k=3$

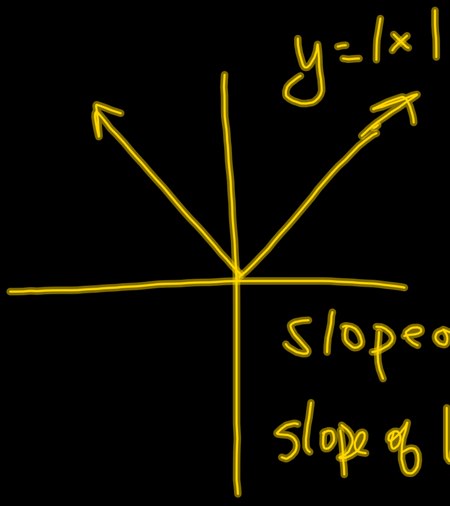
An important concept in calculus involves the concept of **differentiability**. There are several definitions that you need to know:

Definitions

Differentiability at a point: Function $f(x)$ is differentiable at $x = c$ if and only if $f'(c)$ exists. That is, $f'(c)$ is a real number.

Differentiability on an interval: Function $f(x)$ is differentiable on an interval (a,b) if and only if it is diffferentiable for every value of x on the interval (a,b).

Differentiability: Function $f(x)$ is differentiable if and only if it is differentiable at every value of x in its



slope of $|x|$ when $x>0$ is 1

slope of $|x|$ when $x<0$ is -1

The value of derivative as $x\to 0^+$ is 1.

The value of deriv. as $x \to 0^-$ is -1.

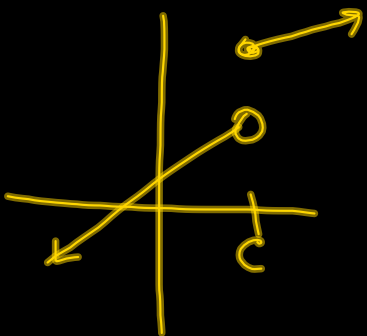
not same

$\therefore$ $|x|$ is not differentiable @ $x=0$.

The absolute value function is continuous on its domain, but it is not differentiable at $x=0$.

Just because a function is continuous on its domain does not mean it is differentiable on its domain.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$



ABCD's

defined at every point and its limit must exist everywhere. That implies the following:

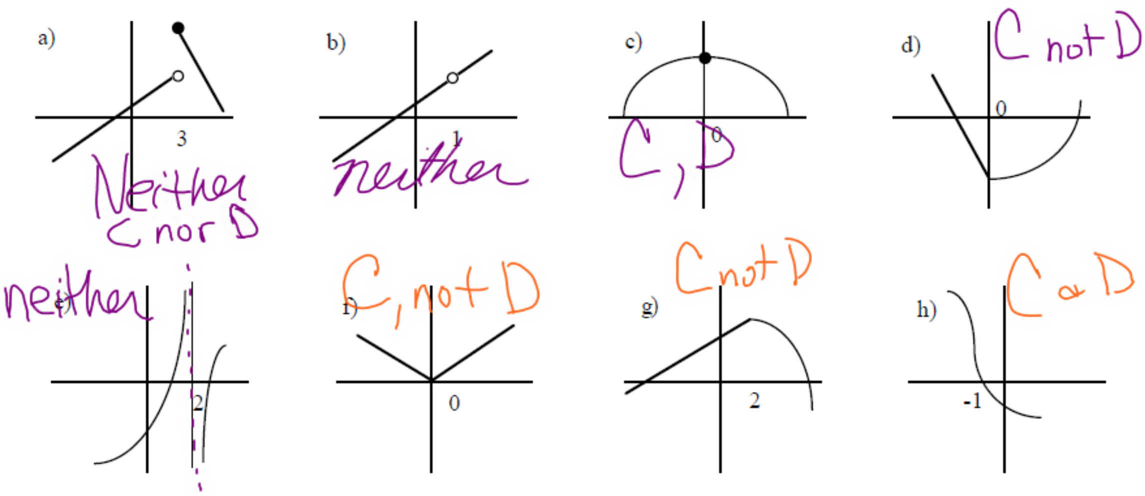
Differentiability implies Continuity, Continuity does not imply Differentiability.

If a function $f(x)$ is differentiable at $x = c$, then it must be continuous also at $x = c$. $\boxed{D \Rightarrow C}$

However, if a function is continuous at $x = c$, it need not be differentiable at $x = c$. $\boxed{\text{Not! } C \Rightarrow D}$

And, if a function is not continuous, then it can't be differentiable at $x = c$.. $\boxed{\text{not } C \Rightarrow \text{ not } D}$

Example: determine whether the following functions are continuous, differentiable, neither, or both at the point.



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i) $f(x)=x^2-6x+1$

C + D on its domain

j) $f(x)=\dfrac{x^2-x-12}{x+3}$

At $x=-3$ discontinuous

k) $f(x)=\sin x$

C+D on its domain

l) $f(x)=\dfrac{\sin x}{x}$

not cont. @ $x=0$

Example 2) Find if $f(x)$ is continuous and/or differentiable at the value of the function where the rule changes. Sketch the function.

a) $f(x)=\begin{cases} x^2-6x+10, x\ge 2 \\ 4-x, x<2 \end{cases}$

b) $f(x)=\begin{cases} x^2+x-3, x\ge -1 \\ -x-4, x<-1 \end{cases}$

$f'(x)=\begin{cases} 2x-6 \\ -1 \end{cases}$

$f'(2)$ top piece is -2 & bottom piece is -1. $\therefore$ $f(x)$ is not diff. @ $x=2$.

First check for continuity:

$f(2)=2^2-6\cdot 2+10$

$\left.\begin{array}{l} f(2)=2 \\ 4-2=2 \end{array}\right\}$ $f(x)$ is continuous @ $x=2$.

$\left.\begin{array}{l} \lim\limits_{x\to 2^-} f(x)=2 \\ \lim\limits_{x\to 2^+} f(x)=2 \end{array}\right\}$ $\lim\limits_{x\to 2} f(x)=2$

$f(2)=2$ same

$\therefore$ $f(x)$ is continuous @ $x=2$.

Example 3) Find the values of a and b that make the function $f(x)$ differentiable.

a) $f(x)=\begin{cases} ax^2+1, x\geq 1 \\ bx-3, x<1 \end{cases}$

b) $f(x)=\begin{cases} ax^3+1, x<2 \\ b(x-3)^2+10, x\geq 2 \end{cases}$

$1^{st} \rightarrow$ make $f(x)$ continuous.

$a\cdot 1^2+1=b\cdot 1-3$

$a+1=b-3$

$\rightarrow a-b=-4$

2^{nd}: derivative must be same

$2ax=b$ when $x=1$

$\rightarrow 2a=b$

$a-2a=-4$

$\boxed{\begin{matrix} a=4 \\ b=8 \end{matrix}}$