AP Calculus AB
Tuesday, October 8, 2013

Quiz Thursday over Trig Derivatives & Function Analysis: 30 minutes

Find relative extrema.

d. $f(x) = -\cos x - \frac{1}{2}x$ on $[0, 2\pi]$

$f'(x) = -1(-\sin x) - \frac{1}{2}$

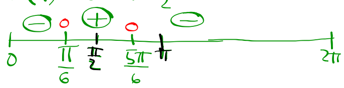
$f'(x) = \sin x - \frac{1}{2}$

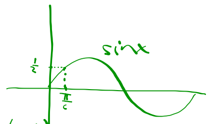
$\sin x - \frac{1}{2} = 0$

$\sin x = \frac{1}{2}$ on $[0, 2\pi]$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$f'(x) = \sin x - \frac{1}{2}$





on $(0, \frac{\pi}{6})$, $\sin x < \frac{1}{2}$

Since $f'(x)$ goes from negative to positive @ $x = \pi/6$, $f(x)$ has a relative minimum @ $x = \frac{\pi}{6}$

Since $f'(x)$ goes from positive to negative at $x = \frac{5\pi}{6}$, $f(x)$ has a relative maximum at $x = \frac{5\pi}{6}$.

The relative minimum is

$f(\frac{\pi}{6}) = -\cos\frac{\pi}{6} - \frac{1}{2}(\frac{\pi}{6})$

$= -\frac{\sqrt{3}}{2} - \frac{\pi}{12}$

The relative maximum is

$f(\frac{5\pi}{6}) = -\cos\frac{5\pi}{6} - \frac{1}{2}(\frac{5\pi}{6})$

$= -(-\frac{\sqrt{3}}{2}) - \frac{5\pi}{12}$

$= \frac{\sqrt{3}}{2} - \frac{5\pi}{12}$

Now, find points of inflection & describe the concavity of f(x).

$f(x) = -\cos x - \frac{1}{2}x$

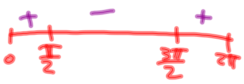
$f'(x) = \sin x - \frac{1}{2}$

$f''(x) = \cos x$

$\cos x = 0$ $[0, 2\pi]$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$f''(x) = \cos x$

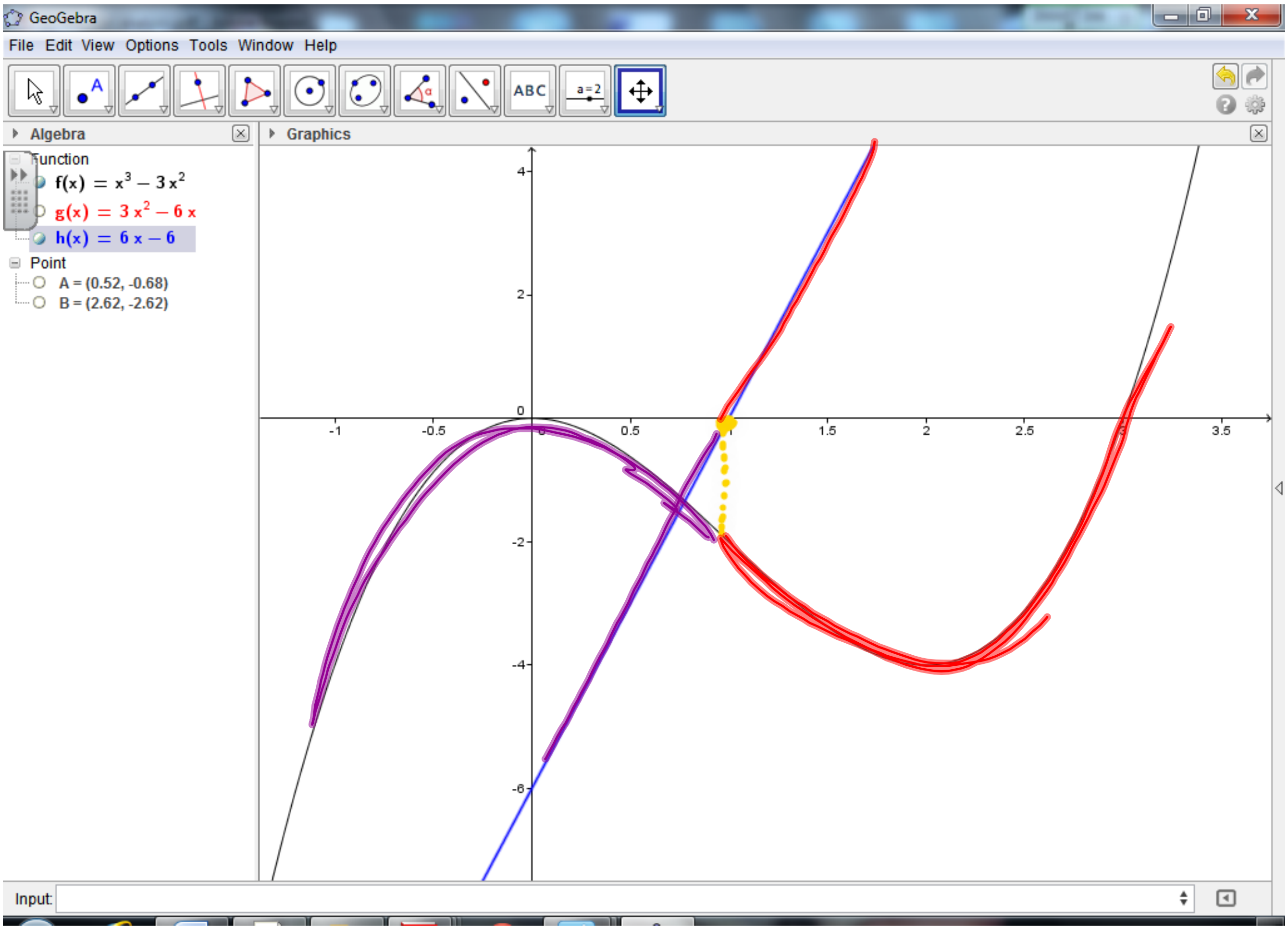


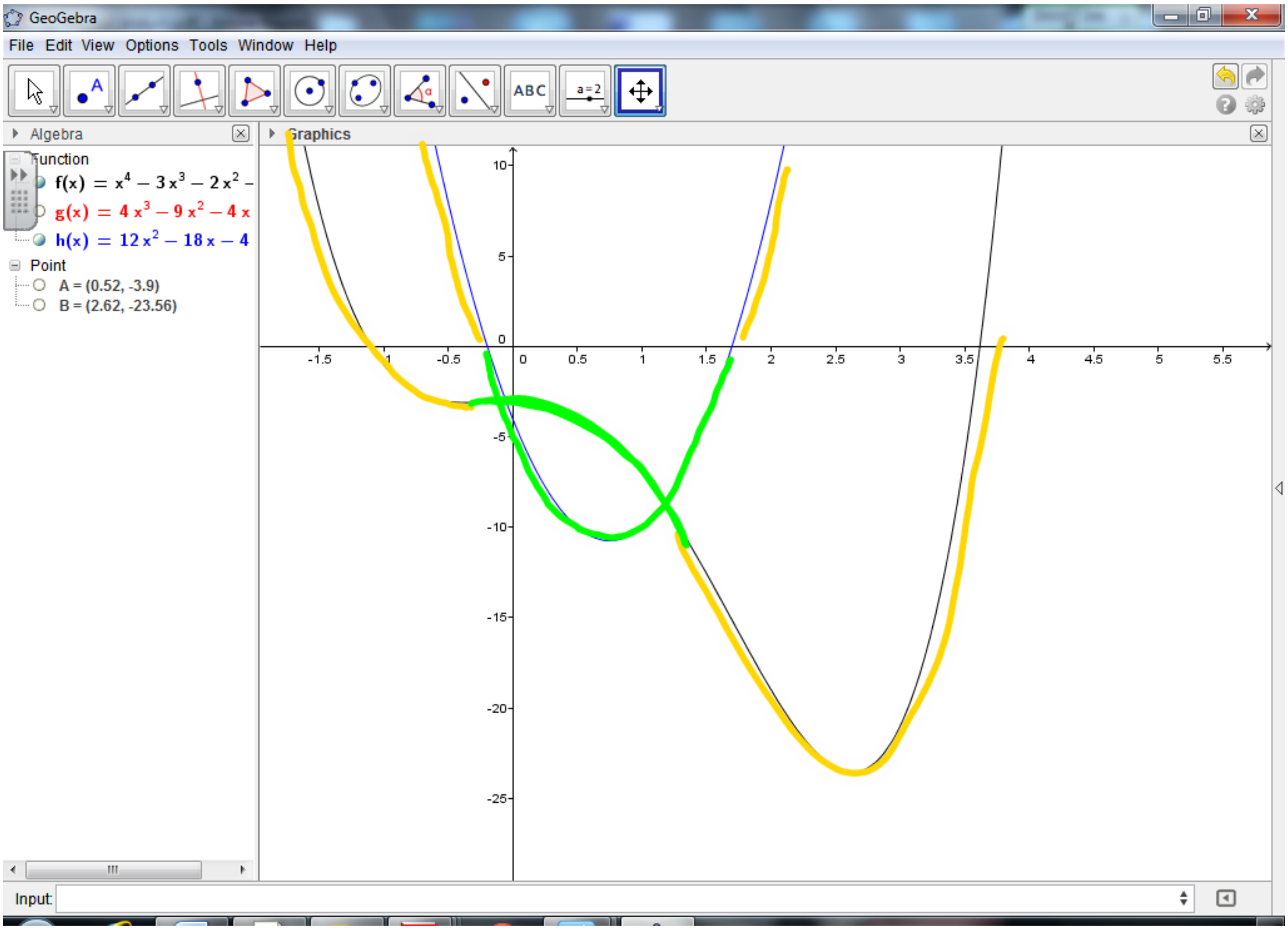
Since $f''(x)$ changes signs at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$, $f(x)$ has points of inflection at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$. Also because $f''(x) > 0$ on $(0, \frac{\pi}{2})$ and $(\frac{3\pi}{2}, 2\pi)$, $f(x)$ is concave up on $(0, \frac{\pi}{2})$ and $(\frac{3\pi}{2}, 2\pi)$. $f''(x) < 0$ on $(\frac{\pi}{2}, \frac{3\pi}{2})$ therefore, $f(x)$ is concave down on $(\frac{\pi}{2}, \frac{3\pi}{2})$.

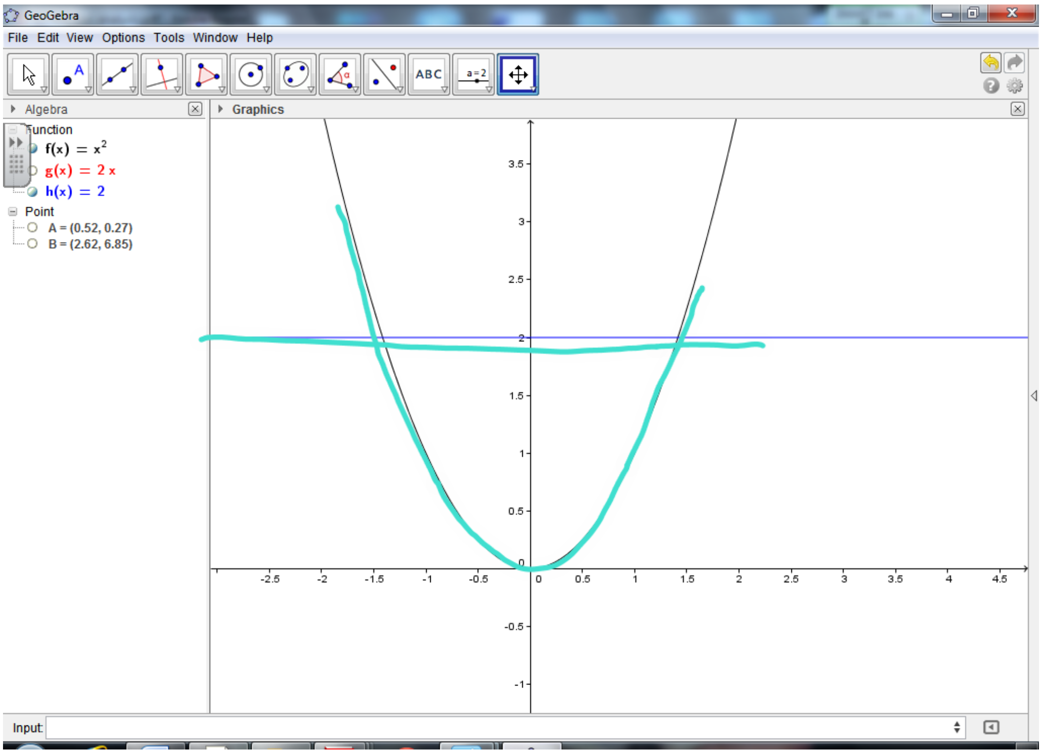
$s(t)$ position

$s'(t)$ velocity

$s''(t)$ acceleration

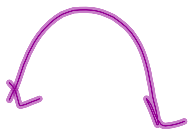

GeoGebra
File Edit View Options Tools Window Help
ABC
a = 2
Algebra
Graphics
Function
$f(x) = x^3 - 3x^2$
$g(x) = 3x^2 - 6x$
$h(x) = 6x - 6$
Point
A = (0.52, -0.68)
B = (2.62, -2.62)
-1
-0.5
0
0.5
1.5
2
2.5
3.5
4
2
-2
-4
-6
Input:


GeoGebra
File Edit View Options Tools Window Help
ABC
a = 2
Algebra
Graphics
Function
f(x) = x^4 − 3 x^3 − 2 x^2 −
g(x) = 4 x^3 − 9 x^2 − 4 x
h(x) = 12 x^2 − 18 x − 4
Point
A = (0.52, -3.9)
B = (2.62, -23.56)
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
10
5
-5
-10
-15
-20
-25
Input



When the second derivative is positive, the original function is concave UP.

When the second derivative is negative, the original function is concave DOWN.



If a curve changes concavity, there is a point of inflection. There is a point of inflection at the place where the second derivative changes signs.

The first derivative finds relative extrema and tells us where the function is increasing or decreasing. The second derivative finds points of inflection and describes the concavity of the original function.