AP Calculus AB
Wednesday, October 30, 2013

HalloweenActivity

Bellwork:CheckanswerstoHWwithsomeone. Writethenumberofany"problem"problemson theboard.

1) $\frac{dy}{dx} = \frac{-1}{4}$

3) $\frac{dy}{dx} = \frac{-5}{4}$

5) $\frac{dy}{dx} = \frac{5}{8}$

7) $\frac{dy}{dx} = \frac{1}{4}$

9) $\frac{dy}{dx} = \frac{1-3x^2y}{x^3-1}$

11) $\frac{dy}{dx} = \cot x \cot y$

13) $\frac{dy}{dx} = \frac{y+1}{1-x}$

$\frac{d^2y}{dx^2} = \frac{2y+2}{(1-x)^2}$

15) $\frac{dy}{dx} = \frac{4-2x}{8y+16}$

	horizontal	vertical
	$(2,0),(2,-4)$	$(6,-2),(-2,-2)$

⑥ $x^2y - xy^2 = -6$ Find $\frac{dy}{dx}$ @ $(2,-1)$.

$x^2y' + y\cdot 2x\cdot x' - \left[x\cdot 2y\cdot y' + y^2\cdot x'\right] = 0$

$x^2y' + 2xy - 2xy\cdot y' - y^2 = 0$

$y'(x^2 - 2xy) = y^2 - 2xy$

$y' = \frac{y^2 - 2xy}{x^2 - 2xy}$

$y'\Big|_{(2,-1)} = \frac{(-1)^2 - 2\cdot 2(-1)}{2^2 - 2\cdot 2(-1)}$

$y'\Big|_{(2,-1)} = \frac{1+4}{4+4} = \frac{5}{8}$

⑦ $\sqrt{xy} = x - 2y$ @ $(4,1)$

$(xy)^{1/2} = x - 2y$

$\frac{1}{2}(xy)^{-1/2}(xy' + yx') = x' - 2y'$

$\frac{xy' + y}{2\sqrt{xy}} = \frac{1 - 2y'}{1}$

$xy' + y = 2\sqrt{xy}(1 - 2y')$

$xy' + y = 2\sqrt{xy} - 4y'\sqrt{xy}$

$xy' + 4y'\sqrt{xy} = 2\sqrt{xy} - y$

$y'(x + 4\sqrt{xy}) = 2\sqrt{xy} - y$

$y' = \frac{2\sqrt{xy} - y}{x + 4\sqrt{xy}}$

y'@$(4,1)$: $= \frac{2\sqrt{4\cdot 1} - 1}{4 + 4\sqrt{4\cdot 1}}$

$= \frac{3}{12} = \boxed{\frac{1}{4}}$

⑨ $x^3y - y = x$

$x^3y' + y \cdot 3x^2 \cdot x' - y' = x'$

$x^3y' + 3x^2y - y' = 1$

$y'(x^3 - 1) = 1 - 3x^2y$

$y' = \frac{1 - 3x^2y}{x^3 - 1}$ OR $\frac{3x^2y - 1}{1 - x^3}$

$5 - 3 = 2$

$3 - 5 = -2$

⑬ $1 - xy = x - y$ find $\frac{d^2y}{dx^2}$ (second derivative)

$-[xy' + yx'] = x' - y'$

$-xy' - y = 1 - y'$

$y' - xy' = y + 1$

$y'(1 - x) = y + 1$

$y' = \frac{y+1}{1-x}$

$y'' = \frac{(1-x)(y') - (y+1)(-x')}{(1-x)^2}$

$y'' = \frac{y'(1-x) - (y+1)(-1)}{(1-x)^2}$

$y'' = \frac{\left(\frac{y+1}{1-x}\right)(1-x) + y + 1}{(1-x)^2}$

$y'' = \frac{y+1+y+1}{(1-x)^2}$

$y'' = \frac{2y+2}{(1-x)^2}$

(15) $x^2+4y^2-4x+16y+4=0$

$2x+8y\cdot y'-4+16y'=0$

$y'(8y+16)=4-2x$

$y'=\frac{4-2x}{8y+16}$

$y'=\frac{2-x}{4y+8}$ → horizontal tan line $2-x=0$

→ vertical tan line $4y+8=0$

HTL: $x=2$ VTL: $y=-2$

Go back to orig. eqn to find points.

HTL:

$2^2+4y^2-4\cdot2+16y+4=0$

$4y^2+16y=0$

$4y(y+4)=0$

$y=0 \quad y=-4$

$(2,0) \quad (2,-4)$

VTL

$x^2+4(-2)^2-4x+16(-2)+4=0$

$x^2+16-4x-32+4=0$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

$x=6 \quad x=-2$

$(6,-2) \quad (-2,-2)$