

Grades...

Bellwork:

1. Graph $x^2 + y^2 = 25$. Plot the point $(4, 3)$.
2. Find the slope of the tangent line to the curve at the point $(4, 3)$ by following the following steps:

First, we can solve for y .
Next, take the derivative:
and simplify
Now plug in the value of $x = 4$.

BUT...we have a +/- in our derivative. Which one is it? Why?

$$x^2 + y^2 = 25$$

$$y^2 = 25 - x^2$$

$$y = \pm \sqrt{25 - x^2}$$

$$y = \pm (25 - x^2)^{1/2}$$

$$y' = \pm \frac{1}{2} (25 - x^2)^{-1/2} (-2x)$$

$$y' = \pm x (25 - x^2)^{-1/2}$$

$$y' = \frac{\pm x}{\sqrt{25 - x^2}}$$

$$y'(4) = \frac{\pm 4}{\sqrt{25 - 4^2}}$$

$$y'(4) = \frac{\pm 4}{3}$$

$$y'(4) = -4/3 \text{ bc of graph}$$

$$\text{Ex. } y + \tan(xy) = 1$$

$$y^3 + y^2x - y + x = 4$$

} These are examples of equations which require a different method for differentiating because we can't solve them for y .

Ex. $x^2 + y^2 = 25$

Take derivative with respect to x of each term - REMEMBER CHAIN RULE!

$$\frac{d}{dx}[x^2] + \frac{d}{dx}[y^2] = \frac{d}{dx}[25]$$

$$2x \cdot \frac{dx}{dx} + 2y \cdot \frac{dy}{dx} = 0$$

Solve for $\frac{dy}{dx}$.

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{2y \frac{dy}{dx}}{2y} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

Find value of $\frac{dy}{dx}$ @ $(4, 3)$.

$$\left. \frac{dy}{dx} \right|_{(4,3)} = \frac{-4}{3}$$

Ex. Find $\frac{dy}{dx}$ for $x^2 - y^2 = 16$ @ $(5, -3)$

Solution:

$$\frac{d}{dx}[x^2] - \frac{d}{dx}[y^2] = \frac{d}{dx}[16]$$

$$2x \cdot \frac{dx}{dx} - 2y \cdot \frac{dy}{dx} = 0$$

$$2x - 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{x}{y}$$

Evaluate @ $(5, -3)$

$$\left. \frac{dy}{dx} \right|_{(5,-3)} = \frac{5}{-3}$$

Ex. Find $\frac{dy}{dx}$ for $xy + y = 8$ @ $(2, 3)$.

Solution:

$$\frac{d}{dx}[xy] + \frac{d}{dx}[y] = \frac{d}{dx}[8]$$

$$x \cdot \frac{dy}{dx} + y \cdot \frac{dx}{dx} + \frac{dy}{dx} = 0$$

$$x \frac{dy}{dx} + y + \frac{dy}{dx} = 0$$

$$x \frac{dy}{dx} + \frac{dy}{dx} = -y$$

$$\frac{dy}{dx}(x+1) = -y$$

$$\frac{dy}{dx} = \frac{-y}{x+1}$$

Evaluate @ $(2, 3)$

$$\left. \frac{dy}{dx} \right|_{(2,3)} = \frac{-3}{2+1} = -1$$

① Find $\frac{dy}{dx}$ for $x^2y + xy^2 = 2x$ @ $(1, 1)$

② Find $\frac{dy}{dx}$ for $y + \sqrt{xy} = 4$ @ $(3, 2)$

③ Find $\frac{dy}{dx}$ for $(x+y)^2 + y = 2$ @ $(0, 1)$

④ Find $\frac{dy}{dx}$ for $x^2 + 4y^2 = 4$ @ $(2, 0)$

⑤ Find $\frac{dy}{dx}$ for $\sin(xy) = 1$.