AP Calculus AB

Thursday, October 25, 2012

Example 9) Find the equation of the tangent line to
$$\sqrt{x} + \sqrt{y} - 1 = y \text{ at } (9,4)$$

$$(x)^{1/2} + y^{1/2} - 1 = y$$

$$\frac{1}{2}x^{-1/2} \cdot \frac{dx}{dx} + \frac{1}{2}y^{-1/2} \cdot \frac{dy}{dx} = \frac{dy}{dx}$$

$$\frac{1}{2}x^{-1/2} = \frac{dy}{dx} - \frac{1}{2}y^{-1/2}\frac{dy}{dx}$$

$$\frac{1}{2}x^{-1/2} = \frac{dy}{dx}\left(1 - \frac{1}{2}y^{-1/2}\right)$$

$$\frac{\frac{1}{2}x^{-1/2}}{1 - \frac{1}{2}y^{-1/2}} = \frac{dy}{dx}$$

@ $(9,4)$

$$\frac{\frac{1}{2}(9)^{-1/2}}{1 - \frac{1}{2}(4)^{-1/2}} = \frac{dy}{dx}\bigg|_{(9,4)}$$

$$\frac{\frac{1}{2} \cdot \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{2}} =$$

$$\frac{\frac{1}{6}}{\frac{3}{4}}$$

$$\frac{1}{6} \cdot \frac{4}{3}$$

$$\boxed{\frac{2}{9}}$$

$$y - 4 = \frac{2}{9}(x - 9)$$

Example 8) Given $x^2 + y^2 = 100$, find $\dfrac{d^2 y}{dx^2}$

$$2x \frac{dx}{dx} + 2y \frac{dy}{dx} = 0$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \boxed{\frac{-x}{y}}$$

$$\frac{d^2 y}{dx^2} = \frac{y \cdot \frac{dx}{dx} - (-x)\left(\frac{dy}{dx}\right)}{y^2}$$

$$\frac{d^2 y}{dx^2} = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$$

$$\frac{d^2 y}{dx^2} = \frac{-y - \frac{x^2}{y}}{y^2}$$

$$= \frac{\dfrac{-y^2 - x^2}{y}}{y^2}$$

$$= \boxed{\frac{-y^2 - x^2}{y^3}}$$