AP Calculus AB
Wednesday, October 24, 2012

Bellwork: Find the equation of the tangent line of x^2 + y^2 = 36 at x = 2. Include a sketch of the curve & the tangent line.

Today's Essential Question: How and when do I perform implicit differentiation?

Finding derivative without solving for y first.

$f(x) = x^2$ @ $x = 2$

EQN of the line

$f(2) = 4$

$f'(x) = 2x$

$f'(2) = 4 \rightarrow$ slope

$y - 4 = 4(x-2)$

Bellwork: $x^2 + y^2 = 36$

$y^2 = 36 - x^2$

$y = \sqrt{36 - x^2}$ and $-\sqrt{36-x^2}$

Find y' to find slope:

$y = (36 - x^2)^{1/2}$

$y' = \frac{1}{2}(36-x^2)^{-1/2}(-2x)$

$y' = \dfrac{-x}{\sqrt{36-x^2}}$

y' @ $x = 2$

$y' = \dfrac{-2}{\sqrt{32}} \rightarrow$ slope of tangent line @ $x=2$

$y = -(36-x^2)^{1/2}$

$\vdots$

$y' = \dfrac{x}{\sqrt{36-x^2}}$

$y' = \dfrac{2}{\sqrt{32}}$

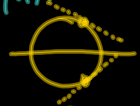
$x^2 + y^2 = 36$

$2^2 + y^2 = 36$

$y^2 = 32$

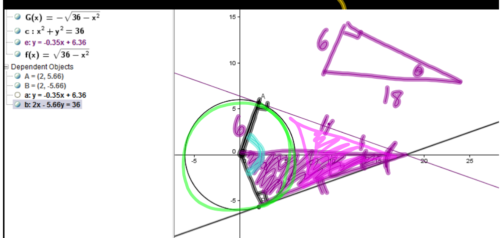
$y = \pm\sqrt{32}$

Two values: $(2, \sqrt{32})$ $(2, -\sqrt{32})$



$y - \sqrt{32} = \dfrac{-2}{\sqrt{32}}(x-2)$

$y + \sqrt{32} = \dfrac{2}{\sqrt{32}}(x-2)$



$\dfrac{\text{Circumf.}}{360°} = \dfrac{\text{arclength}}{\text{central } \angle}$

$$x^2+y^2=36$$

Find deriv. with respect to x.

$$2x\cdot\frac{dx}{dx}+2y\cdot\frac{dy}{dx}=0$$

$$2x+2y\cdot\frac{dy}{dx}=0$$

Solve for $\frac{dy}{dx}$.

$$2y\frac{dy}{dx}=-2x$$

$$\frac{dy}{dx}=\frac{-x}{y}$$

Slope @ $x=2$?

We already found y

$(2,\sqrt{32})$

$$\left.\frac{dy}{dx}\right|_{x=2}=\frac{-2}{\sqrt{32}}$$

Ex. Find $\frac{dy}{dx}$ for $xy+y=8$ @ $(2,3)$.

Solution:

$$x\frac{dy}{dx}+y\cdot\frac{dx}{dx}+\frac{dy}{dx}=0$$

Solve for $\frac{dy}{dx}$.

$$x\frac{dy}{dx}+y+\frac{dy}{dx}=0$$

$$\frac{dy}{dx}(x+1)=-y$$

$$\frac{dy}{dx}=\frac{-y}{x+1}$$

$$\left.\frac{dy}{dx}\right|_{(2,3)}=\frac{-3}{3}=-1$$