

AP Calculus AB

Monday, October 22, 2012

12) Give the value of x where the function

$$f(x) = x^3 - 9x^2 + 24x + 4$$

has a local maximum.

a) 4 $f'(x) = 3x^2 - 18x + 24$

b) -2 $3x^2 - 18x + 24 = 0$

c) 2 $x^2 - 6x + 8 = 0$
 $(x-4)(x-2) = 0$

d) -4

e) 3

$x = 2, x = 4$ $x = 2$

10) Determine the concavity of the graph of

$f'(x) = 3\cos x + 8(\cos x) \cdot (-\sin x)$ $f(x) = 3\sin(x) + 4(\cos(x))^2$

$f'(x) = 3\cos x - 4(2\cos x \sin x)$ at $x = \pi$

$f'(x) = 3\cos x - 4\sin 2x$

$f''(x) = -3\sin x - 4(\cos 2x) \cdot 2$

$f''(x) = -3\sin x - 8\cos 2x$

$f''(\pi) = -3\sin \pi - 8\cos 2\pi$

$f''(\pi) = -8$

OR

$f'(x) = 3\cos x - 8\cos x \sin x$

$f'(x) = \cos x (3 - 8\sin x)$

$f''(x) = \cos x (-8\cos x) + (3 - 8\sin x)(-\sin x)$

$f''(\pi) = \cos \pi (-8\cos \pi) + (3 - 0)(-\sin \pi)$

$f''(\pi) = -1(-8)(-1) + 0$

$f''(\pi) = -8$

When an object is subjected to gravity, its position function is given by $s(t) = -16t^2 + v_0t + s_0$, where t is measured in seconds, $s(t)$ is measured in feet, v_0 is the initial velocity (velocity at $t = 0$) and s_0 is the initial position (position at $t = 0$). The formula is given by $s(t) = -4.9t^2 + v_0t + s_0$ if $s(t)$ is measured in meters.

From our original $s(t) = -16t^2 + v_0t + s_0$, we can calculate the velocity function $v(t) = -32t + v_0$ and the acceleration function $a(t) = -32$. This is the acceleration due to gravity on earth.

$$s(t) = -4.9t^2 + v_0t + s_0 \quad \text{m}$$

$$v(t) = -9.8t + v_0 \quad \text{m/s}$$

$$a(t) = -9.8 \quad \text{m/s}^2$$

When an object is thrown upward, it is subjected to gravity. We are usually interested how high the particle reaches and how fast it is going when it impacts the ground or water. Let us analyze what these mean:

When an object reaches its maximum height, what is its velocity? 0 ft/sec

When an object hits the ground, what is its final position? 0

So to find the maximum height of an object, set $v(t) = 0$, solve for t , and find $s(t)$.

So, to find the velocity of an object when it hits the ground, set $s(t) = 0$, solve for t , and find $v(t)$.

Example 4) A projectile is launched vertically upward from ground level with an initial velocity of 112 ft/sec.

a. Find the velocity and speed at $t = 3$ and $t = 5$ seconds. b. How high will the projectile rise? c. Find the speed of the projectile when it hits the ground.

$$s(t) = -16t^2 + v_0t + s_0$$

$$v_0 = 112 \text{ ft/sec}$$

$$s_0 = 0 \text{ ft}$$

$$s(t) = -16t^2 + 112t \rightarrow \text{position eqn}$$

Find velocity & speed.

$$v(t) = -32t + 112$$

$$v(3) = -32(3) + 112 \quad v(5) = -32(5) + 112$$

$$v(3) = 16 \text{ ft/sec} \quad v(5) = -48 \text{ ft/sec}$$

$$\text{Speed} = |v(t)| \quad \text{Speed} = 48 \text{ ft/sec}$$

Speed @ 3 sec is 16 ft/sec bc $v(5) < 0$, the particle is moving down @ $t = 5$

$$v(t) = 0 \rightarrow \text{time Max. ht.}$$

$$-32t + 112 = 0$$

$$32t = 112$$

$$t = \frac{112}{32} \text{ sec.}$$

$$t = 3.5 \text{ sec}$$

$$s(3.5) = -16(3.5)^2 + 112(3.5)$$

$$s(3.5) = 196 \text{ ft max ht.}$$

To find velocity when particle hits ground, set $s(t) = 0$. Plug that $t \rightarrow v(t)$.

$$s(t) = 0$$

$$-16t^2 + 112t = 0$$

$$-16t(t - 7) = 0$$

$$t = 7$$

$$v(7) = -32(7) + 112$$

$$v(7) = -112 \text{ ft/sec}$$

Speed when particle hits ground @ 112 ft/sec

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c) How high did the rock go? d) How long did it take the rock to reach half its maximum height?

e) How long was the rock aloft? e) Find the rock's speed when hitting the moon.

Example 7) A ball is $v_0 = 0$ dropped from the top of the Washington Monument which is $s_0 = 555$ feet high.

a) How long will it take for the ball to hit the ground? b) Find the ball's speed at impact.

Example 8) Paul has bought a ticket on a special roller coaster at an amusement park which moves in a straight line. The position $s(t)$ of the car in feet after t seconds is given by: $s(t) = -t^3 + 120t^2$, $0 \leq t \leq 120$

a) Find the velocity and acceleration of the roller coaster after t seconds? b) When is the roller coaster stopped?

c) When is Paul speeding up and slowing down? d) Where is Paul at critical times of his ride?