APCalculusAB
Monday,January7,2013

Anyquestionsontheexam?
Unit21MMM

GooverHW...youwillbeputtingproblemsontheboard

RetakesbyFriday.

[redacted]

13. $\int 3\sqrt[3]{x^2}\,dx$

14. $\int (x-5)^2\,dx$

15. $\int 4(3x-2)^3\,dx$

16. $\int \frac{x^3-4x-1}{2x^3}\,dx$

17. $\int t^2(3+t)^2\,dt$

18. $\int \frac{(3x-2)^2}{\sqrt{x}}\,dx$

[redacted]

19. $\int \frac{3\cos x}{5}\,dx$

20. $\int (1-6\cos x)\,dx$

21. $\int \left(\frac{1}{x^2}-\sin x\right)dx$

22. $\int (\sec^2 t+\cos t+1)\,dt$

23. $\int (\sin^2 x+\cos^2 x)\,dx$

24. $\int \frac{\sin x}{1-\sin^2 x}\,dx$

23 $\int 1\,dx = x+C$

24

$$\int \frac{\sin x}{\cos^2 x}\,dx$$

$$= \int \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\,dx$$

$$= \int \tan x\sec x\,dx$$

$$= \sec x + C$$

If you were given the statement that $\frac{dy}{dx} = 4x$, we can cross multiply to get $dy = 4x\,dx$. We can now integrate each side of the equation to get $\int dy = \int 4x\,dx$. From there, we can solve for y.

Differentials

The original statement $\frac{dy}{dx} = 4x$ is called a differential equation (DEQ). In a differential equation, you are given a statement about the derivative of y, $\frac{dy}{dx}$. Your goal is to solve for y. We have done so with the exception of the $+C$, the constant of integration. So we have a *general solution* of the DEQ. But suppose we were told that if $x = 0$, then $y = 5$. From there we can solve for C and we will thus have the *specific solution* of the DEQ. Let's do so.

Example 18) Solve the differential equation.

$$f'(x) = 3x - 1, f(2) = 3$$

Example 19) Solve the differential equation.

$$f'(x) = x^2 - 2x + 2, f(3) = -1$$

$$\frac{dy}{dx} = 4x$$

$$\int dy = \int 4x\,dx$$

$$y = \frac{4x^2}{2} + C$$

$$y = 2x^2 + C$$

$x = 0$ when $y = 5$, find C.

$$5 = 2 \cdot 0^2 + C$$

$$C = 5$$

specific solution: $y = 2x^2 + 5$

$$f'(x) = x^2 - 2x + 2$$
$$f(3) = -1$$

$$\frac{dy}{dx} = x^2 - 2x + 2$$

$$\int dy = \int (x^2 - 2x + 2)\,dx$$

$$y = \frac{x^3}{3} - x^2 + 2x + C$$

$$f(3) = -1$$

$$-1 = 9 - 9 + 6 + C$$

$$C = -7$$

$$f(x) = \frac{x^3}{3} - x^2 + 2x - 7$$

Ex. $\left.\begin{aligned} f'(x) &= 3x - 1 \\ f(2) &= 3 \end{aligned}\right\}$ Solve the differential equation.

Remember

$$f'(x) = \frac{dy}{dx}$$

$$\frac{dy}{dx} = 3x - 1$$

cross multiply

$$\int dy = \int (3x - 1)\,dx$$

$$y = \frac{3x^2}{2} - x + C$$

Remember $\rightarrow y = f(x)$.

$$f(x) = \frac{3x^2}{2} - x + C$$

Given: $f(2) = 3$

$$3 = \frac{3 \cdot 2^2}{2} - 2 + C$$

$$3 = 4 + C$$

$$C = -1$$

$$f(x) = \frac{3x^2}{2} - x - 1$$

Example 21) Solve the differential equation.

$f''(x) = 2x, f'(-5) = 30, f(2) = -1$

$f''(x) = 2x \quad f'(-5) = 30 \quad f(2) = -1$

$\frac{d^2y}{dx^2} = 2x$

$\int d^2y = \int 2x\,dx^2$

$dy = x^2 dx + C$

use $f'(-5) = 30$

$\frac{dy}{dx} = x^2 + C$

$f'(x) = x^2 + C$

$30 = (-5)^2 + C$

$C = 5$

$f'(x) = x^2 + 5$

$\frac{dy}{dx} = x^2 + 5$

$\int dy = \int (x^2+5)dx$

$y = \frac{x^3}{3} + 5x + C$

$f(2) = -1$

$-1 = \frac{8}{3} + 10 + C$

$-\frac{3}{3} = \frac{8}{3} + \frac{30}{3} + C$

$-\frac{41}{3} = C$

$f(x) = \frac{x^3}{3} + 5x - \frac{41}{3}$

Example 22) Given that the graph of $f(x)$ passes through the point (1, 6) and that the slope of its tangent line at $(x, f(x))$ is $2x+1$, find $f(6)$.

$f'(x) = 2x + 1$

$\frac{dy}{dx} = 2x + 1$

$\int dy = \int (2x+1)dx$

$y = x^2 + x + C$

$f(x) = x^2 + x + C$

Given: $(1,6)$ on $f(x)$

$6 = 1^2 + 1 + C$

$C = 4$

$f(x) = x^2 + x + 4$

asked for $f(6)$.

$f(6) = 6^2 + 6 + 4$

$f(6) = 46$

Wed the 9th → summative assessment (80%)

Unit 21

3rd Quarter