APCalculusA
Friday,January18,2013

Morewithareaunderthecurve

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Bellwork: (You may use your calculator)

2. A particle moves along the x-axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(a) $a(t) = v'(t)$

$a(2) = 1.588$

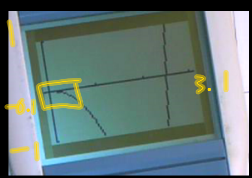
$v(2) = -2.728$

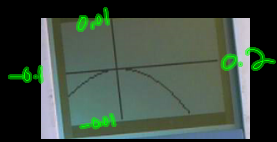
Since velocity and acceleration have different signs, the particle is slowing down.

(b) When the velocity changes signs, the particle will be changing direction.

$v(t) = 0$

$-(t+1)\sin\left(\frac{t^2}{2}\right) = 0$





$-(t+1) = 0$

$t = -1$

$-t-1 = 0$

$-t = 1$

$\cancel{t = -1}$

$\underline{0 < t < 3}$

$\sin\left(\frac{t^2}{2}\right) = 0$

$\sin \boxed{0, \pi, 2\pi} = 0$

$\frac{t^2}{2} = 0, \pi, 2\pi$

$\frac{t^2}{2} = 0$

$\cancel{t = 0}$

$\frac{t^2}{2} = \pi$

$t^2 = 2\pi$

$t = \pm\sqrt{2\pi}$

$t = \sqrt{2\pi}$

$t \approx 2.507$

$\frac{t^2}{2} = 2\pi$

$t^2 = 4\pi$

$t = \pm\sqrt{4\pi}$

$t = \sqrt{4\pi}$

$\cancel{t \approx 3.545}$

$v(2) = -2.728$

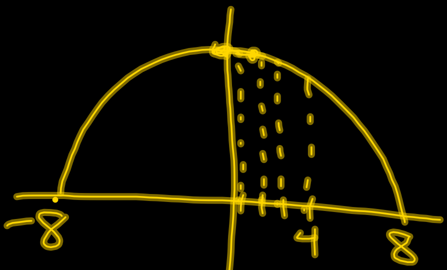
$v(\sqrt{2\pi}) = 0$

$v(3) = 3.910$

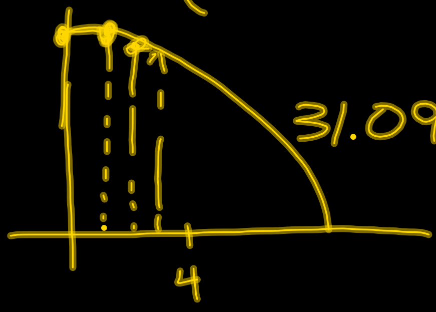
The particle changes direction @ $t = \sqrt{2\pi}$.

It only changes direction once on $0 < t < 3$.

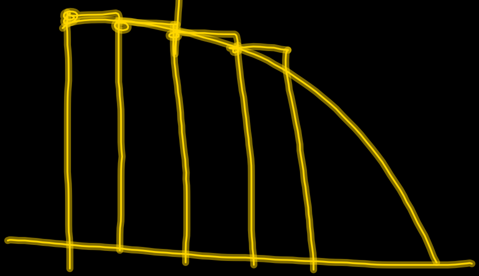
$f(x) = \sqrt{64 - x^2}$

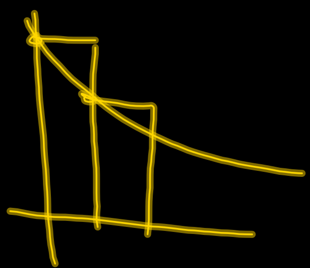


$\text{area} = 1\left(f(0) + f(1) + f(2) + f(3)\right)$



31.099 un^2



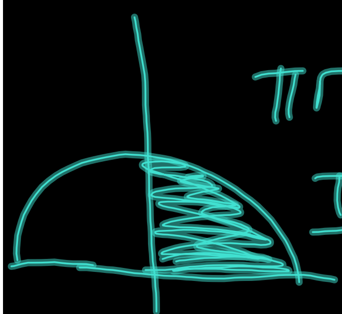


8 rect → 0 to 8

$\text{area: } \underbrace{\frac{8-0}{8}}_{\text{width/base of rect.}}\left[f(1) + f(2) + \dots + f(8)\right]$

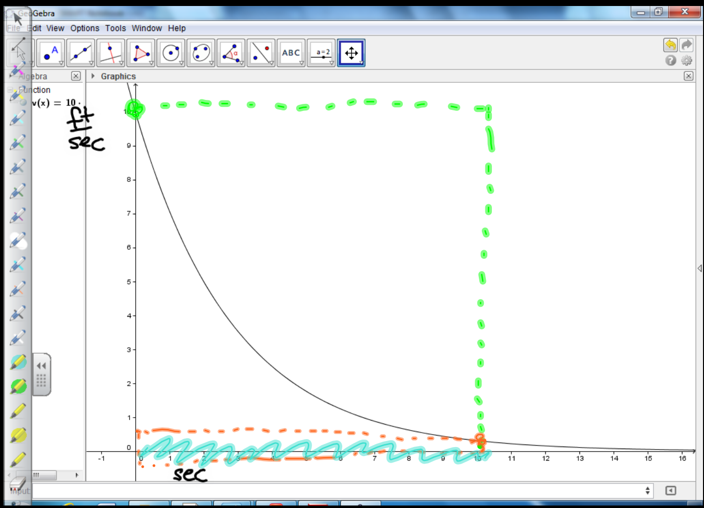
4.564

45.437



πr^2

$\frac{\pi 8^2}{4} \approx 50.265$



$$v(t) = 10 \cdot 2^{-\frac{t}{2}}$$

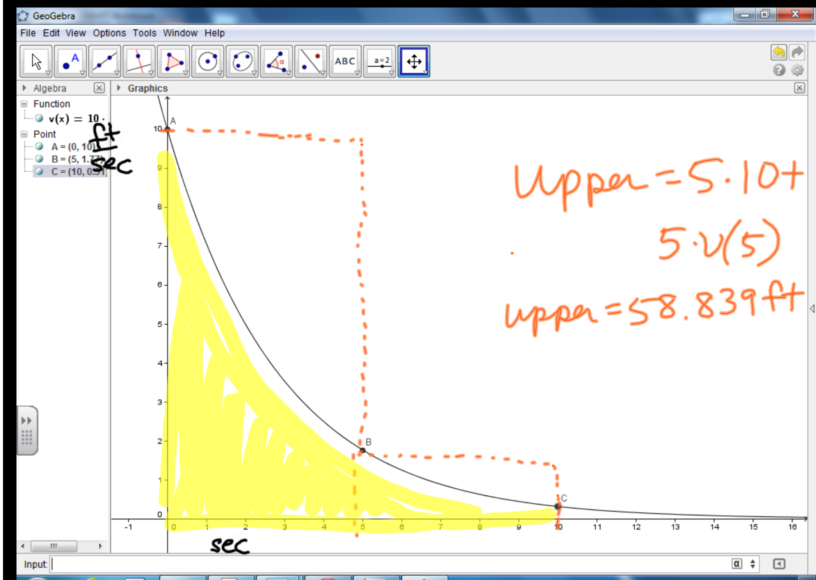
$$v(10) = 10 \cdot 2^{-5}$$

$$v(10) = \frac{10}{32}$$

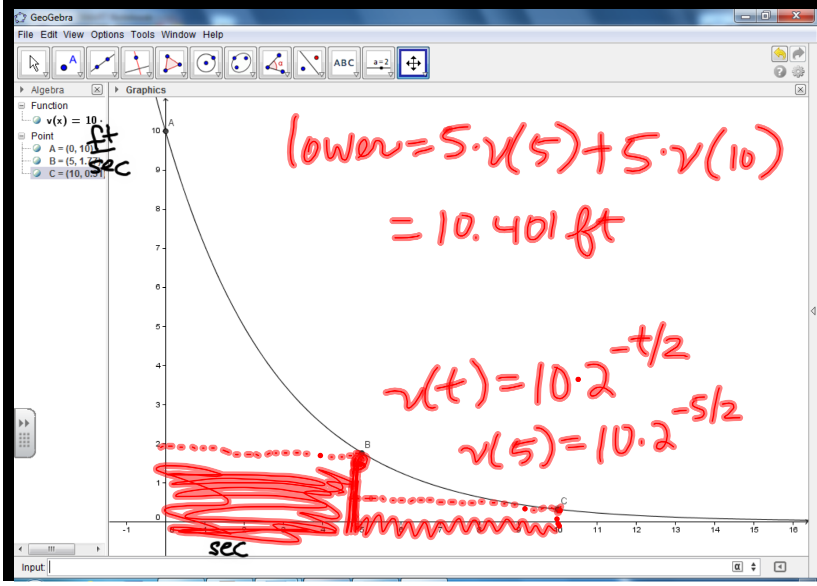
$$\frac{10}{32} \cdot 10 = 3.125\,ft$$

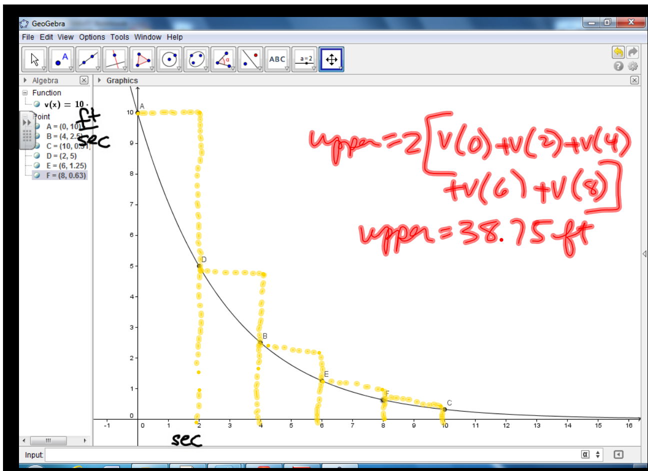
Green rectangle : $10 \frac{ft}{sec} \cdot 10\,sec = 100\,ft$

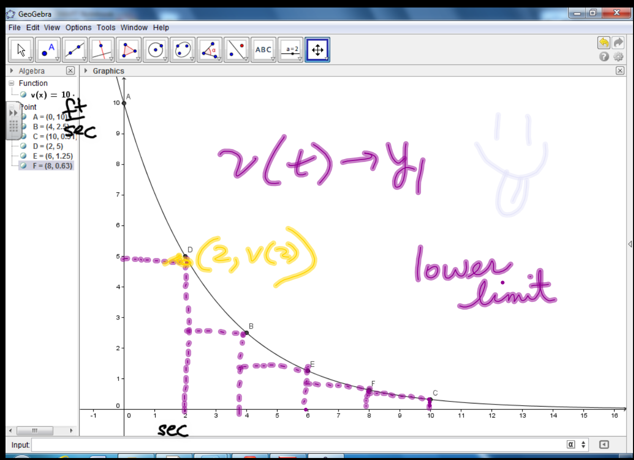
Orange : $\frac{10}{32} \frac{ft}{sec} \cdot 10\,sec = 3.125\,ft$



$$10 \cdot 2^{-5/2}$$





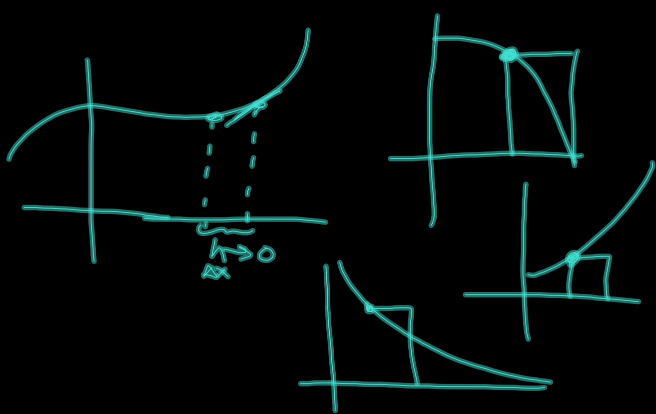


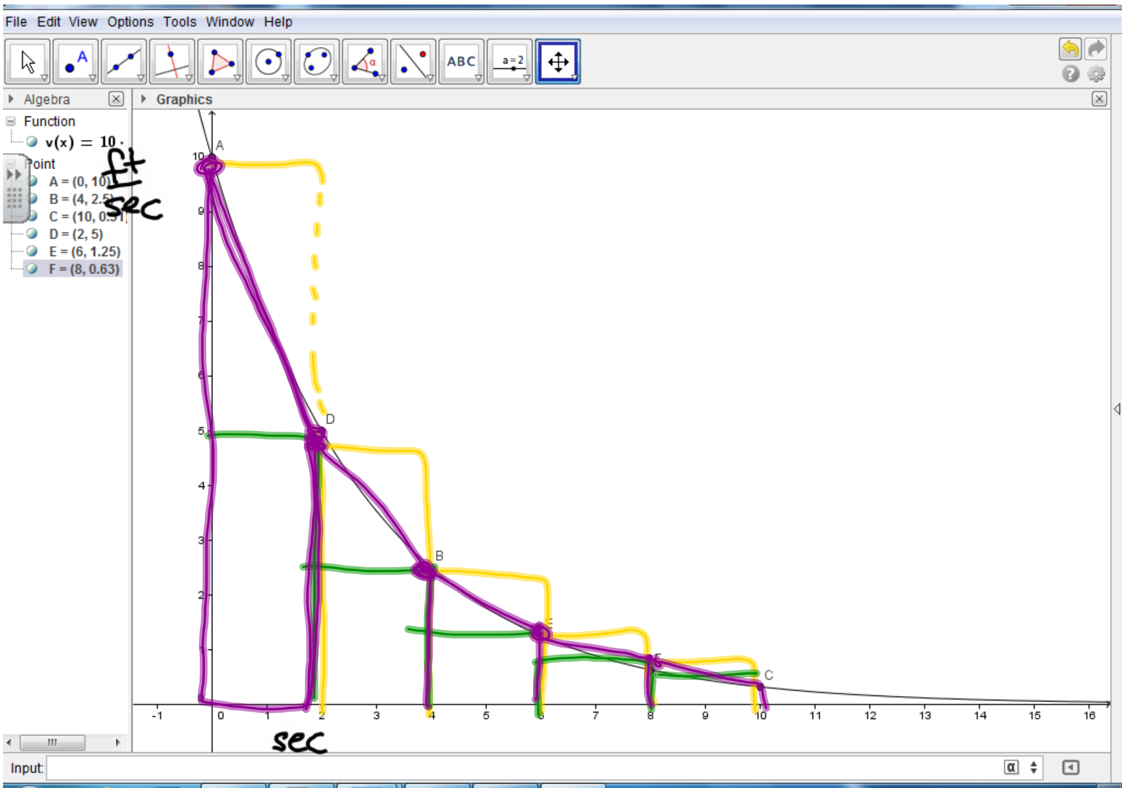
$$\text{lowerlimit} = 2[v(2)+v(4)+v(6)+v(8)+v(10)]$$
$$= 19.375 \text{ ft}$$

10 rectangles of equal width:

$$\text{lower} = 1[v(1)+v(2)+v(3)+\cdots+v(10)]$$
$$\text{lower} = 23.388 \text{ ft}$$

$$\text{upper} = 1[v(0)+v(1)+v(2)+\cdots+v(9)]$$
$$\text{upper} = 33.075 \text{ ft}$$





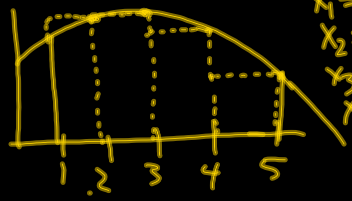
Trapezoid: $\frac{1}{2}\cdot 2[v(0)+v(2)+v(2)+v(4)+v(4)+v(6)+v(6)+v(8)+v(8)+v(10)]$

$$\text{Trap} = \frac{1}{2}\cdot 2[v(0)+2\cdot v(2)+2\cdot v(4)+2\cdot v(6)+2\cdot v(8)+v(10)]$$

area using traps: 29.0625 ft

Area Under the Curve...please read. It is a summary of what we've been doing.

Riemann Sums...



$x_0 = 1$
$x_1 = 2$
$x_2 = 3$
$x_3 = 4$
$x_4 = 5$

Right

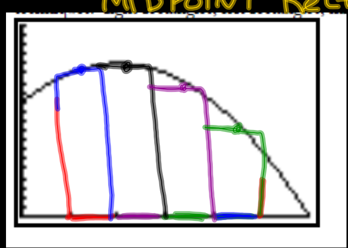
$A \approx b\left(f(x_1) + f(x_2) + f(x_3) + f(x_4)\right)$

Left

$A \approx b\left[f(x_0) + f(x_1) + f(x_2) + f(x_3)\right]$

$$\left.\begin{aligned} A_{left} &\approx b\sum_{i=0}^{3} f(x_i) \\ A_{right} &\approx b\sum_{i=1}^{4} f(x_i) \end{aligned}\right\} \text{Riemann Sums}$$

MIDPOINT Rectangles

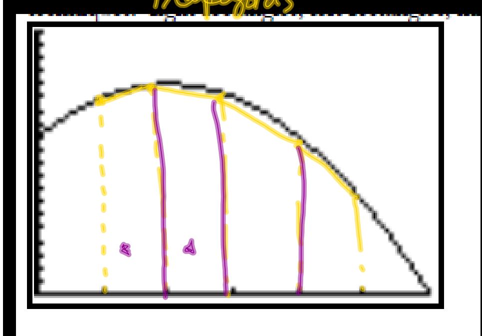


$\text{Area} \approx 1\cdot\left[f(1.5) + f(2.5) + f(3.5) + f(4.5)\right]$

Generalize mdpt. rectangles

$A \approx b\cdot\left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + f\left(\frac{x_2+x_3}{2}\right) + f\left(\frac{x_3+x_4}{2}\right)\right]$

Trapezoids



$\frac{1}{2}\cdot base\left[f(x_0) + 2\cdot f(x_1) + 2\cdot f(x_2) + 2\cdot f(x_3) + f(x_4)\right]$

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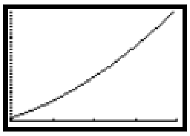
Let's try one: Let $f(x) = x^2 - 3$. We want to find the area under the curve using 8 rectangles/trapezoids from $x = 2$ to $x = 6$. First, let's draw it. Note that the curve is completely above the axis. If it dips below, the method changes slightly.



The drawing of the curve is helpful, but not necessary.

Since there are 8 rectangles, and we are finding the area between $x = 2$ and $x = 6$, the base is $\frac{6-2}{8} = \frac{1}{2}$

Let's complete the chart:

i	x_i	$f(x_i)$
0	2	1
1	2.5	13/4
2	3	6
3	3.5	37/4
4	4	13
5	4.5	65/4
6	5	22
7	5.5	109/4
8	6	33

Right Rectangles

2 2.25 2.5

$\frac{5}{2} = \frac{25}{4} - \frac{12}{4}$

So, the right rectangle formula gives __________________________

the left rectangle formula give __________________________

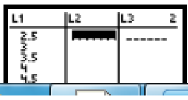
the trapezoid formula gives $1 + 2\left(\frac{13}{4}\right) + 2(6) + 2\left(\frac{37}{4}\right) + 2(13) + \cdots + 33$

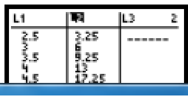
Note that the chart will not give you the midpoint formula. Let's do it here:

$f(2.25) +$ __________________________

The calculator can generate this chart. Let's use right rectangles. Go to $\boxed{\text{STAT}}$ EDIT and clear out L1 and L2.

Place your x_i in L1. It will look like this: Now L2 contains $f(x_i)$. Since your function is in Y1, use





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