

Solve. State the domain first.

①  ~~$\frac{3}{x} = \frac{4}{x-1}$~~

②  $\frac{x}{4} = \frac{9}{x}$

$x \neq 0, 1$

$4x = 3(x-1)$

$4x = 3x - 3$

~~$x = -3$~~

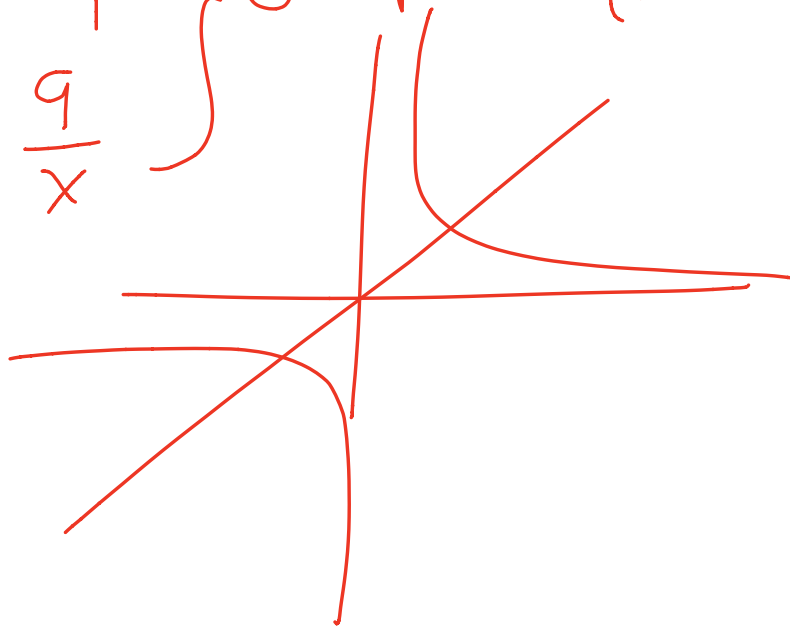
check:  $\frac{3}{-3} \stackrel{?}{=} \frac{4}{-3-1}$   
 $-1 = -1 \checkmark$

$$\frac{x}{4} = \frac{9}{x}$$

$$y_1 = \frac{x}{4}$$

$$y_2 = \frac{9}{x}$$

Graph — look for intersection



3/24 How do I solve rational equations analytically?

1<sup>st</sup> → State excluded values!

2<sup>nd</sup> → Write as a proportion - then cross multiply.

Solve:  
EX.  $\frac{9}{28} + \frac{3}{x+2} = \frac{3}{4}$

$x \neq -2$

$$\frac{3}{x+2} = \frac{13}{28} - \frac{9}{28}$$

$$\frac{3}{x+2} = \frac{21}{28} - \frac{9}{28}$$

$$\frac{3}{x+2} = \frac{12}{28}$$

$$\frac{3}{x+2} = \frac{3}{7}$$

$$x+2=7$$

$$x=5$$

Solve

$$\frac{5}{24} + \frac{2}{3-x} = \frac{1}{4}$$

Excluded Value(s):  $x \neq 3$

$$\frac{2}{3-x} - \frac{61}{64} - \frac{5}{24}$$

$$\frac{2}{3-x} = \frac{1}{24}$$

$$3-x = 48$$

$$-x = 45$$

$$x = -45$$

Ex. Solve:

$$x + \frac{x^2 - 5}{x^2 - 1} = \frac{x^2 + x + 2}{x + 1}$$

$$x \neq 1, -1$$

$$x = \frac{(x-1)(x^2 + x + 2)}{(x-1)(x+1)} - \frac{x^2 - 5}{x^2 - 1}$$

$$x = \frac{(x-1)(x^2 + x + 2)}{x^2 - 1} - \frac{x^2 - 5}{x^2 - 1}$$

$$X = \frac{(x-1)(x^2+x+2) - (x^2-5)}{x^2-1}$$

$$X = \frac{x^3 + \cancel{x^2} + 2x - \cancel{x^2} - x - 2 - x^2 + 5}{x^2-1}$$

$$X = \frac{x^3 - x^2 + x + 3}{x^2-1}$$

$$x^3 - x^2 + x + 3 = x(x^2 - 1)$$

$$x^3 - x^2 + x + 3 = x^3 - x$$

$$-x^3 \quad +x \quad -x^3 + x$$


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$$-1(-x^2 + 2x + 3 = 0)$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x-3=0 \quad x+1=0$$

$x=3$        ~~$x=-1$  Extraneous~~

Solve

$$\frac{x+1}{x-3} = 4 - \frac{12}{x^2 - 2x - 3}$$

$x \neq 3, -1$        $\underbrace{x^2 - 2x - 3}_{(x-3)(x+1)}$

$$\left(\frac{x+1}{x+1}\right) \frac{x+1}{x-3} + \frac{12}{(x-3)(x+1)} = 4$$

$$\frac{x^2 + 2x + 1 + 12}{(x-3)(x+1)} = 4$$



$$(x-3)(x+1)$$

$$x^2 + 2x + 13 = 4(x^2 - 2x - 3)$$

$$x^2 + 2x + 13 = 4x^2 - 8x - 12$$

$$0 = 3x^2 - 10x - 25$$

$$0 = (3x+5)(x-5)$$

$$3x+5=0$$

$$x = -\frac{5}{3} \checkmark$$

$$x-5=0$$

$$x=5 \checkmark$$

②  $\frac{x}{4} = \frac{9}{x}$   $x \neq 0$

$$y = \frac{x}{4} = \frac{1}{4}x \rightarrow \text{line}$$

$$x^2 = 36$$

$$x = 6, -6$$

$$\left. \begin{array}{l} y_1 = \frac{1}{4}x \\ y_2 = \frac{9}{x} \end{array} \right\} \begin{array}{l} \text{Find} \\ \text{Intersection} \end{array}$$

Ex. Solve:

$$\frac{2}{x-1} + 4 = \frac{5}{x-1}$$

Solution:

$$x \neq 1$$

$$4 = \frac{5}{x-1} - \frac{2}{x-1}$$

~~$$4 = \frac{3}{x-1}$$~~

$$4x - 4 = 3$$

$$4x = 7$$

$$x = \frac{7}{4}$$



Ex. Solve:

$$\left(\frac{x+3}{x+3}\right) \frac{2}{x-3} + \frac{1}{\frac{x^2-9}{(x+3)(x-3)}} = 3$$

Solution: Excluded values  
 $x \neq 3, -3$

$$\frac{2(x+3)}{(x+3)(x-3)} + \frac{1}{(x+3)(x-3)} = 3$$

$$\frac{2x+6+1}{(x+3)(x-3)} = 3$$

$$\frac{2x+7}{(x+3)(x-3)} = \frac{3}{1}$$

$$2x+7 = 3(x^2-9)$$

$$2x+7=3x^2-27$$

$$0=3x^2-2x-34$$

$$x = \frac{2 \pm \sqrt{4 - 4 \cdot 3(-34)}}{6}$$

$$x = \frac{2 \pm \sqrt{412}}{6}$$

$$x = \frac{2 \pm 2\sqrt{103}}{6}$$

$$x = \frac{1 \pm \sqrt{103}}{3}$$

$$x = \frac{1 + \sqrt{103}}{3}$$

$$\approx 3.716$$

$$x = \frac{1 - \sqrt{103}}{3}$$

$$x \approx -3.049$$

Solve:

$$\textcircled{1} \frac{7}{24} + \frac{5}{x-1} = \frac{1}{6}$$

$$\textcircled{2} x + \frac{x^2 - 5}{x^2 - 1} = \frac{x^2 + x + 2}{x + 1}$$