1. $\int_{-2}^{2}|1-x^2|\,dx$ $\int_{-2}^{-1}-(1-x^2)+\int_{-1}^{1}1-x^2+\int_{1}^{2}-(1-x^2)$

$0=1-x^2$

$x=\pm 1$

$-\;|\;+\;|\;-$

$-1 \quad 1$

$-\left(x-\frac{x^3}{3}\right)\Big|_{-2}^{-1}+x-\frac{x^3}{3}\Big|_{-1}^{1}-\left(x-\frac{x^3}{3}\right)\Big|_{1}^{2}$

$-\left(-\frac{2}{3}-\frac{2}{3}\right)+\left(\frac{2}{3}+\frac{2}{3}\right)-\left(-\frac{2}{3}-\frac{2}{3}\right)$

$-\left(-\frac{4}{3}\right)+\left(\frac{4}{3}\right)-\left(-\frac{4}{3}\right)=\boxed{4}$

2. $\int_{0}^{5}|x-4|\,dx$

$x-4=0$

$x=0$

$-\;|\;+$

4

$\boxed{\int_{0}^{4}(4-x)\,dx}+\boxed{\int_{4}^{5}(x-4)\,dx}$

$\left[4x-\frac{x^2}{2}\right]_0^4+\left[\frac{x^2}{2}-4x\right]_4^5$

$16-8-0-0+\frac{25}{2}-20-(8-16)$

$16-8+\frac{25}{2}-20-8+16=\boxed{\frac{17}{2}}$

3. $\int_{0}^{4}|x-\sqrt{x}|\,dx$

$x-\sqrt{x}=0$

$x=\sqrt{x}$

$x^2=x$

only at 1, $\cancel{0}$

$-\;|\;+$

1

$\int_{0}^{1}(\sqrt{x}-x)\,dx+\int_{1}^{4}(x-\sqrt{x})\,dx$

$\left[\frac{2x^{3/2}}{3}-\frac{x^2}{2}\right]_0^1+\left[\frac{x^2}{2}-\frac{2x^{3/2}}{3}\right]_1^4$

$\frac{2}{3}-\frac{1}{2}-0-0+8-\frac{16}{3}-\left(\frac{1}{2}-\frac{2}{3}\right)$

$=\boxed{3}$

4. $\int_{0}^{4}|9-x^2|\,dx$

$9-x^2=0$

$-x^2=-9$

$x^2=9$

$x=\pm 3$

$+\;|\;-$

3

$\int_{0}^{3}(9-x^2)\,dx+\int_{3}^{4}(x^2-9)\,dx$

$\left[9x-\frac{x^3}{3}\right]_0^3+\left[\frac{x^3}{3}-9x\right]_3^4$

$27-9-0-0+\frac{64}{3}-36-(9-27)$

$=\boxed{\frac{64}{3}}$